Lesson 1-1
Patterns and Inductive Reasoning

Vocabulary
Inductive reasoning is reasoning based on patterns you observe.

Example
Finding and Using a Pattern
Find the next two terms in each sequence:

a. Monday, Tuesday, Wednesday, ,...

b. , ,4 ,7 ,11 ,16 ,22 ,29 ,37 ,...

c. , ,4 ,7 ,11 ,16 ,22 ,29 ,37 ,...

Quick Check

1. Make a conjecture about the sum of the first 25 counting numbers. Use your calculator to verify your conjecture.

2. Make a conjecture about the sum of the first 35 odd numbers. Use your calculator to verify your conjecture.

Answers may vary. Sample:

For the first 25 counting numbers, or (1 + 25)2 = 625.

For the first 35 odd numbers is 35 2, or 1225.

Lesson 1-2
Drawings, Nets, and Other Models

Vocabulary
An isometric drawing of a three-dimensional object shows a corner view of the figure drawn on isometric dot paper.

An orthographic drawing is the top view, front view, and right-side view of a three-dimensional figure.

Quick Check

1. Make an orthographic drawing from this isometric drawing:

   Front

   Top

   Right

2. Draw a different net for this box. Show the dimensions in your diagram.

   Front:

   Top:

   Right:

   Bottom:

   Left:

   Right:

Answers may vary. Example:

Orthographic Drawing

Make an orthographic drawing of the isometric drawing at right.

Orthographic drawings show the depth of a figure. An orthographic drawing shows three views of the three-dimensional figure.

Think of the sides of the square base as hinges and “unfold” the figure. Label the net with its dimensions:

Draw a different net for this box. Show the dimensions in your diagram.

Answers may vary. Example:
Lesson 1-3
Points, Lines, and Planes

Vocabulary and Key Concepts

Postulate 1-1
Through any two points there is exactly one line.

Postulate 1-2
If two lines intersect, they intersect in exactly one point.

Postulate 1-3
Through any three noncollinear points there is exactly one plane.

Postulate 1-4
A point is a location. A line is a series of points that extends in two opposite directions, and a plane is the set of all points.

Collinear points are points that lie on the same line.

Lesson Objectives
- Identify segments and rays
- Recognize parallel lines
- Understand basic terms of geometry
- Name a line that is parallel to plane ABC
- Name a line that is parallel to plane ABC
- Name a line that is parallel to plane ABC
- Name a line that is parallel to plane ABC

Examples

Quick Check

1. Use the figure in Example 1. and form a line. Are opposite rays? Explain.

2. Use the diagram to the right. How do they not have the same endpoints?
Lesson 1-6 Measuring Angles

Name_____________________________________ Class____________________________ Date________________

Lesson Objectives
✓ Find the measure of angles
✓ Identify special angle pairs

Vocabulary and Key Concepts

Postulate 5-2: Postulate Postulate
Let \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) be the opposite rays in a plane.

Postulate 5-3: Angle Addition Postulate
If point \( B \) is in the interior of \( \angle AOC \), then

\[ \angle AOB + \angle BOC = \angle AOC \]

Examples:

1. **Using the Segment Addition Postulate**
   If \( AB = 25 \), then find \( AD \) and \( DC \).
   Use the Segment Addition Postulate (Postulate 1-6) to write an equation.
   \[ AD + DC = AB \]
   \[ AD + 25 = 100 \]
   Subtract from each side.
   \[ AD = 75 \]

2. **Finding Lengths**
   \( M \) is the midpoint of \( EF \).
   Find \( RM, MT, \) and \( RT \).
   Use the definition of midpoint to write an equation.
   \[ BM = \frac{1}{2} EF \]
   \[ BM = \frac{1}{2} (10 + 20) \]
   \[ BM = 15 \]

Quick Check:

1. **Using Angles**
   Name the angle at right in four ways.
   The name can be the vertex of the angle.
   \[ \angle B \]

2. **Critical Thinking**
   Would it be correct to name any of the angles \( \angle EFG \) ? Explain.
   No, 3 angles have \( F \) as a vertex, so you need more information in the name to distinguish them from one another.

Local Standards: ____________________________________
Lesson 1-7 Basic Constructions

**Vocabulary.**

- **Background Line:** A line used to draw circles and arcs.

**Key Concepts.**

- **Perpendicular Bisector:** A geometric tool used to draw circles and parts of circles called arcs.

**Examples.**

1. **Constructing Congruent Segments**
   - **Construction:** To construct congruent segments.
   - **Steps:**
     1. Draw a ray with endpoint A to B.
     2. Construct congruent segments AB and CD.
     3. Use a compass and a straightedge to construct the perpendicular bisector of the segment.

2. **Finding the Midpoint**
   - **Problem:** Find the midpoint of a segment in the coordinate plane.
   - **Solution:**
     1. Use the Midpoint Formula. Let (x1, y1) and (x2, y2) be points on the segment.
     2. Substitute for x and y.

---

Lesson 1-8 The Coordinate Plane

**Vocabulary.**

- **Coordinate Plane:** A two-dimensional plane formed by the intersection of a horizontal and a vertical number line called axes.

**Key Concepts.**

- **Distance Formula:** The distance d between two points (x1, y1) and (x2, y2) is given by the formula: 
  \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

- **Midpoint Formula:** The midpoint M of a segment with endpoints (x1, y1) and (x2, y2) is given by the formula: 
  \[ M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

**Examples.**

1. **Finding the Midpoint**
   - **Problem:** Find the midpoint of a segment with endpoints (4, 6) and (8, 9).
   - **Solution:**
     1. Use the Midpoint Formula. Let (x1, y1) = (4, 6) and (x2, y2) = (8, 9).
     2. Substitute for x and y.

---

Lesson Objectives

1. **Measuring Physical Attributes**
   - **Topic:** Geometry
   - **NAEP 2005 Strand:** Measurement

2. **Relationships Among Geometric Figures**
   - **Topic:** Geometry
   - **NAEP 2005 Strand:** Geometry
Lesson 2-1
Conditional Statements

Lesson Objectives
- Recognize conditional statements.
- Write converses of conditional statements.

Vocabulary and Key Concepts

- Conditional Statements and Converses
  - Statement
  - Example
  - Symbolic Form
  - You read it
  - Conditional: If an angle is straight, then its measure is 180°.
  - p → q
  - If an angle is straight, then its measure is 180°.
  - Converse: If the measure of an angle is 180°, then it is a straight angle.
  - q → p
  - The measure of an angle is 180°, then it is a straight angle.

- A conditional is of the form if-then.
- The hypothesis is the part that follows if in an if-then statement.
- The conclusion is the part of an if-then statement (conditional) that follows then.
- The truth value of a statement is true or false, according to whether the statement is true or false, respectively.

Examples
- Identifying the hypothesis and the conclusion: Identify the hypothesis and conclusion.

  Hypothesis: Two lines are parallel.

  Conclusion: The lines are coplanar.

Quick Check
1. Identify the hypothesis and the conclusion of the conditional statement:
   If p, then q.
   Hypothesis: p
   Conclusion: q

2. Write the converse of the following conditional:
   If two lines are not parallel and do not intersect, then they are skew.
   If two lines are skew, then they are not parallel and do not intersect.

Find the circumference of a circle with a radius of 18 m in terms of π.

Area = πr²

Postulate 1-10
If two figures are congruent, then they have the same area.

Postulate 1-10
The area of a region is the sum of the areas of its non-overlapping parts.

Finding Circumference
- Example: Finding Circumference: Given a radius of 18 m, find the circumference in terms of π.

C = 2πr

C = 2π(18)

C = 36π m

C ≈ 113.0973 m

Find the circumference of a circle with a diameter of 18 m to the nearest tenth.

C = πd

C = π(18)

C = 18π m

C ≈ 56.5 m

Quick Check
1. a. Find the circumference of a circle with a radius of 18 m in terms of π.

2. b. Find the circumference of a circle with a diameter of 18 m to the nearest tenth.

Vocabulary and Key Concepts

- Finding Area: A square with side length s.

Area = s²

- Rectangle with base b and height h.

Area = bh

- Circle with radius r.

Area = πr²

- Perimeter of rectangles and circles.

P = 2l + 2w

P = 2πr

- Postulate 1-9

Local Standards: ____________________________________
Topic: NAEP 2005 Strand: Measuring
Local Standards: ____________________________________
Topic: NAEP 2005 Strand: Physical Attributes

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Lesson 2-3 Deductive Reasoning

Objectives
- Use the Law of Detachment
- Use the Law of Syllogism

Key Concepts

**Law of Detachment**
If a conditional is true and its hypothesis is true, then its conclusion is true.

**Law of Syllogism**
If and are true statements, then is a true statement.

**Deductive Reasoning**
A process of reasoning logically from given facts to a conclusion.

Examples
1. **Using the Law of Detachment**
   - If a conditional is true and its hypothesis is true, then its conclusion is true.
   - Example:
     
   - The conditional is true, and the hypothesis is true.
   - Therefore, the conclusion is true.

2. **Using the Law of Syllogism**
   - If and are true statements, then is a true statement.
   - Example:
     
   - The first conditional is true, and the hypothesis of the second conditional is true.
   - Therefore, the conclusion of the second conditional is true.

Quick Check
1. Suppose that a mechanic begins work on a car and finds that the car will not start. Can the mechanic conclude that the car has a dead battery? Explain.
   - No, there could be other things wrong with the car, such as a faulty starter.

2. If a baseball player is a pitcher, then that player should not pitch a complete game two days in a row. Vladimir Nuñez is a pitcher. On Monday, he pitches a complete game on Tuesday.
   - Answers may vary. Sample: Vladimir Nuñez should not pitch a complete game two days in a row.

3. If a number ends in 4, then it is divisible by 2. If a number ends in 6, then it is divisible by 2. A number divisible by 2 is also divisible by 4. What can you conclude?
   - Answers may vary. Sample: A number divisible by 4 is divisible by 2.
Lesson 2-4

Reasoning in Algebra

Lesson Objectives

- Write proportions and use them to solve problems involving similar figures
- Use algebraic reasoning to solve problems involving proportions

Key Concepts

Properties of Equality

- **Addition Property of Equality**: If \( a = b \), then \( a + c = b + c \)
- **Subtraction Property of Equality**: If \( a = b \), then \( a - c = b - c \)
- **Multiplication Property of Equality**: If \( a = b \), then \( ac = bc \)
- **Division Property of Equality** (positive): If \( a = b \) and \( c \neq 0 \), then \( \frac{a}{c} = \frac{b}{c} \)
- **Reflexive Property of Equality**: \( a = a \)
- **Symmetric Property of Equality**: If \( a = b \), then \( b = a \)
- **Transitive Property of Equality**: If \( a = b \) and \( b = c \), then \( a = c \)
- **Substitution Property of Equality**: If \( a = b \), then \( a \) may replace \( b \) in any expression.

Properties of Congruence

- **Reflexive Property of Congruence**: \( a \cong a \)
- **Symmetric Property of Congruence**: If \( a \cong b \), then \( b \cong a \)
- **Transitive Property of Congruence**: If \( a \cong b \) and \( b \cong c \), then \( a \cong c \)

Examples

- **Using Properties of Equality and Congruence**
  - Given: \( \triangle ABC \) and \( \triangle DEF \)
  - \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), \( \angle C \cong \angle F \)
  - Prove: \( \triangle ABC \cong \triangle DEF \)

Quick Check

1. Fill in each missing reason.

   - Given: \( \angle 1 \) and \( \angle 2 \)
   - Prove: \( \angle 1 \cong \angle 2 \)

   - Reasons: 1. \( m\angle 1 = m\angle 2 \) (Vertical Angles Theorem)

   - \( \angle 1 \) and \( \angle 2 \) are vertical angles.

   - Use the Vertical Angles Theorem.

Lesson 2-5

Proving Angles Congruent

Lesson Objectives

- Prove and apply theorems about angles
- Use algebraic reasoning to solve problems involving congruence

Vocabulary and Key Concepts

- **Theorem 2-1**: Vertical Angles Theorem
  - Vertical angles are congruent.
  - \( \angle 1 \cong \angle 2 \) and \( \angle 3 \cong \angle 4 \)

- **Theorem 2-2**: Congruent Supplements Theorem
  - If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.
  - If \( \angle A \cong \angle B \) and \( \angle B \cong \angle C \), then \( \angle A \cong \angle C \)

- **Theorem 2-3**: Congruent Complements Theorem
  - If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.
  - If \( \angle A \cong \angle B \) and \( \angle B \cong \angle C \), then \( \angle A \cong \angle C \)

- **Theorem 2-4**: Angle Congruence Theorem
  - If two angles are congruent and supplementary, then each is a right angle.

- **Theorem 2-5**: Adjacent Angles Theorem
  - If two angles are supplementary, then they form a linear pair.
  - If \( \angle 1 \) and \( \angle 2 \) are supplementary, then \( \angle 1 \cong \angle 2 \)

Examples

- **Using the Vertical Angles Theorem**
  - Given: \( \angle 1 \) and \( \angle 2 \)
  - \( \angle 1 \cong \angle 2 \)

Quick Check

1. Given: \( \angle 1 \cong \angle 2 \) and \( \angle 2 \cong \angle 3 \)
   - Prove: \( \angle 1 \cong \angle 3 \)

   - **Steps**
     - From given: \( \angle 1 \cong \angle 2 \) and \( \angle 2 \cong \angle 3 \)
     - \( \angle 1 \cong \angle 2 \) and \( \angle 2 \cong \angle 3 \) implies \( \angle 1 \cong \angle 3 \) by transitive property of congruence.
Lesson 3-1: Properties of Parallel Lines

**Lesson Objectives**
- Identify angles formed by two lines and a transversal.
- Prove and use properties of parallel lines.

**Vocabulary and Key Concepts**
- **Corresponding angles** are angles that lie in the same relative position at each intersection.
- **Alternate interior angles** are nonadjacent interior angles that lie on the same side of the transversal.
- **Alternate exterior angles** are nonadjacent exterior angles that lie on the opposite side of the transversal.
- **Vertical angles** are angles that are opposite each other.

**Postulate 3-1: Corresponding Angles Postulate**
If a transversal intersects two parallel lines, then corresponding angles are congruent.

**Theorem 3-1: Alternate Interior Angles Theorem**
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

**Theorem 3-2: Same-Side Interior Angles Theorem**
If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

**Postulate 3-2: Converse of the Corresponding Angles Postulate**
If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.

**Theorem 3-3: Converse of the Alternate Interior Angles Theorem**
If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

**Theorem 3-4: Converse of the Same-Side Interior Angles Theorem**
If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

**Theorem 3-5: Converse of the Alternate Exterior Angles Theorem**
If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.

**Theorem 3-6: Converse of the Same-Side Exterior Angles Theorem**
If two lines and a transversal form same-side exterior angles that are supplementary, then the two lines are parallel.

A flow proof uses arrows to show the logical connections between the statements.

**Reasons** are written below the statements.

---

Lesson 3-2: Proving Lines Parallel

**Lesson Objectives**
- Use a transversal in proving lines parallel.
- Use the diagram from Example 1. Which lines, if any, must be parallel if \( m\angle 1 = m\angle 3 \) and \( m\angle 2 = m\angle 4 \)? Explain.

**Vocabulary and Key Concepts**
- **Corresponding angles** are angles that lie in the same relative position at each intersection.
- **Alternate interior angles** are nonadjacent interior angles that lie on the same side of the transversal.
- **Alternate exterior angles** are nonadjacent exterior angles that lie on the opposite side of the transversal.
- **Vertical angles** are angles that are opposite each other.

**Postulate 3-2: Converse of the Corresponding Angles Postulate**
If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.

**Theorem 3-1: Alternate Interior Angles Theorem**
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

**Theorem 3-2: Same-Side Interior Angles Theorem**
If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

A flow proof uses arrows to show the logical connections between the statements.

**Reasons** are written below the statements.

---

Examples

1. Applying Properties of Parallel Lines: In the diagram of Lafayette Regional Airport, the black segments are runways and the gray areas are taxiways and terminal buildings. Compare \( \overline{tK} \) and the angle vertical to \( \overline{tK} \). Classify the angles as alternate interior angles, same-side interior angles, or corresponding angles.

   The angle vertical to \( \overline{tK} \) is between the runway segments. \( \overline{tK} \) is between the runway segments and on the opposite side of the transversal runway. Because same-side interior angles are not adjacent and lie between the lines on opposite sides of the transversal, \( \overline{tK} \) and the angle vertical to \( \overline{tK} \) are same-side interior angles.

2. Finding Measures of Angles: In the diagram of right, \( f \) and \( g \). Find \( m\angle 1 \) and \( m\angle 2 \).

   Because \( \angle 3 \) and \( \angle 5 \) are corresponding angles, \( m\angle 3 = m\angle 5 \). Because \( \angle 1 \) and \( \angle 2 \) are adjacent angles that form a straight angle, \( m\angle 1 + m\angle 2 = 180^\circ \) by the Angle Addition Postulate. In the equation \( m\angle 1 + m\angle 2 = 180^\circ \), subtract \( m\angle 1 \) from each side to find \( m\angle 2 = 180^\circ - m\angle 1 \).
Lesson 3-3
Parallel and Perpendicular Lines

Key Concepts

Theorem 3-8
If two plane angles are congruent to the same angle, then they are congruent to each other.

Theorem 3-9
If two lines are parallel, then they are perpendicular to the same line.

Theorem 3-10
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

Theorem 3-11
In a plane, if two lines are perpendicular to one of two parallel lines, then they are parallel to the other.

Examples

1. No; answers may vary. Sample:

2. Find the value of \( x \) for which \( \angle Z \) is a right angle and \( \angle X \) is \( 62° \).

\( \angle Z = 90° \), then \( 90° - 62° = 28° \).\n
Quick Check

1. Critical Thinking: In a plane, if two lines form congruent angles with a third line, must the lines be parallel? Draw a diagram to support your answer.

2. Find the value of \( x \) for which \( \angle Z \) is a right angle.

\( Z = 90° \), then \( 90° - 3x - 15° = 60° \).

Lesson 3-4
Parallel Lines and the Triangle Angle-Sum Theorem

Vocabulary and Key Concepts

Theorem 3-12: Triangle Angle-Sum Theorem
The sum of the measures of the angles of a triangle is \( 180° \).

Theorem 3-13: Triangle Exterior Angle Theorem
The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

Examples

1. Applying the Triangle Angle-Sum Theorem: Find \( \angle Z \).

\( \angle Z = 180° - 72° - 56° = 52° \).

2. Applying the Triangle Exterior Angle Theorem: Explain what happens to the angle formed by the back of the chair and the armrest if you make a lounge chair recline more.

Quick Check

1. \( \triangle MNP \) is a right angle and \( \angle X \) is \( 56° \) and \( \angle Y \) is \( 38° \). Find \( m \angle P \).

2. Critical Thinking: Is it true that if two acute angles of a triangle are congruent, then the triangle must be a right triangle? Explain.
Geometry: All-In-One Answers Version B (continued)

### Lesson 3-5
#### The Polygon Angle-Sum Theorems

**Lesson Objectives**
- Classify polygons
- Find the sum of the measure of the interior and exterior angles of a polygon

**Vocabulary and Key Concepts**

**Theorem 3-14: Polygon Angle-Sum Theorem**
The sum of the measures of the interior angles of a polygon is $180(n - 2)$ where $n$ is the number of sides of the polygon.

**Theorem 3-15: Polygon Exterior Angle-Sum Theorem**
The sum of the measures of the exterior angles of a polygon, one at each vertex, is $360^\circ$.

**Examples**

1. **Sum of Interior Angles**
   - For the pentagon, the sum of the interior angles is $180(5 - 2) = 540^\circ$.

2. **Exterior Angle Sum**
   - The sum of the exterior angles is $360^\circ$.

### Lesson 3-6
#### Lines in the Coordinate Plane

**Lesson Objectives**
- Graph lines given their equations
- Write equations of lines

**Vocabulary**

- **Point-Slope Form**
  - The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where $(x_1, y_1)$ is a point on the line and $m$ is the slope.

**Examples**

1. **Graphing Using Intercepts**
   - To graph $2x + 3y = 6$:
     - Find the $y$-intercept: Substitute $x = 0$ to get $y = 2$.
     - Find the $x$-intercept: Substitute $y = 0$ to get $x = 3$.
     - Plot the intercepts $(0, 2)$ and $(3, 0)$ and draw the line.

2. **Equation of a Line**
   - Write an equation of the line that contains the points $(-5, 3)$ and $(2, -1)$.
   - Use the point-slope form: $y - y_1 = m(x - x_1)$.
   - $y - 3 = \frac{-1 - 3}{2 - (-5)}(x - (-5))$.
   - Simplify: $y = \frac{-4}{7}x + \frac{23}{7}$.
Lesson 3-7
Slopes of Parallel and Perpendicular Lines

Lesson Objectives
- Identify parallel lines
- Identify perpendicular lines

Key Concepts
- Slopes of Parallel Lines
  - Two nonvertical lines are parallel if their slopes are equal.
  - Any two vertical lines are parallel.
- Slopes of Perpendicular Lines
  - The slopes of two lines are negative reciprocals of each other.
- Determining Whether Lines are Parallel
  - If two lines are parallel, their slopes are equal.
- Determining Whether Lines are Perpendicular
  - If two lines are perpendicular, the product of their slopes is -1.

Example
1. **Determining Whether Lines are Parallel**
   
   **Example:** Are the lines $y = -5x + 4$ and $x + 5y = 4$ parallel? Explain.
   
   **Solution:** The equation $x + 5y = 4$ is not in slope-intercept form. To determine if the lines are parallel, rewrite the equation in slope-intercept form:
   
   
   
   
   
   
   
   The line $x + 5y = 4$ has slope $\frac{-1}{5}$.

   The line $y = -5x + 4$ has slope $-5$.

   The lines are not parallel because their slopes are not equal.

Quick Check
1. Are the lines $y = 5x + 4$ and $2x + 4y = 9$ parallel? Explain.
   - Yes; Each line has slope $\frac{5}{2}$ and the y-intercepts are different.

2. **Finding Slopes for Perpendicular Lines**
   
   **Example:** Find the slope of a line perpendicular to $y = 2x - 1$.
   
   **Solution:** The slope of a line perpendicular to $y = 2x - 1$ is $\frac{-1}{2}$.

   Multiply each side by $\frac{-1}{2}$.

Lesson 3-8
Constructing Parallel and Perpendicular Lines

Lesson Objectives
- Construct parallel lines
- Construct perpendicular lines

Key Concepts
- Constructing Parallel Lines
  - Use a straightedge to draw a line.
  - Construct so that it is the same distance from point A as it is from point B.

Example
1. **Parallel From a Point to a Line**
   
   **Example:** Examine the diagram at right. Explain how to construct a line parallel to $AB$. Construct the angle.
   
   **Solution:** Use the method learned for constructing congruent angles.

   Step 1: With the compass point on point $A$, draw an arc that intersects the sides of $\angle BAD$.

   Step 2: With the same compass setting, put the compass point on point $B$ and draw an arc.

   Step 3: Put the compass point below point $N$ where the arc intersects line $AB$. Keeping the same compass setting, put the compass point above point $N$ where the arc intersects line $AB$. Draw an arc to locate a point.

   Step 4: Use a straightedge to draw the line through the points you located and point $N$.

Quick Check
1. Use Example 1. Explain why lines $c$ and $d$ must be parallel.

   If corresponding angles are congruent, the lines are parallel by the converse of the Corresponding Angles Postulate.

2. **Perpendicular From a Point to a Line**
   
   **Example:** Examine the diagram. At what special point does $\perp \overline{AB}$ exist? Explain.
   
   **Solution:** Point $D$ is the same distance from point $A$ as it is from point $B$ because both arcs were made with the same compass setting.

   This means that $\perp \overline{AB}$ exists at $D$. and that $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.
Examples.

**Lesson Objective**
- Naming Congruent Parts
  - Congruent polygons are polygons that have corresponding sides congruent and corresponding angles congruent.

**Examples.**

**Lesson 4-1 Daily Notetaking Guide**

**Finding Congruent Triangles**
- Can you conclude that \( \triangle ABC \cong \triangle CDE \)?
- List the corresponding vertices in the same order.
  - If \( \triangle ABC \cong \triangle CDE \), then \( \angle BAC \cong \angle ECD \).
  - The statement \( \triangle ABC \cong \triangle CDE \) is true.
  - Notice that \( \angle M \cong \angle N \) and \( \angle A \cong \angle J \).
  - Using Theorem 4–1, you can conclude that \( \triangle ABC \cong \triangle CDE \).
  - Since all of the corresponding sides and angles are congruent, the triangles are congruent. The correct way to state this is \( \triangle ABC \cong \triangle CDE \).

**Quick Check**
1. \( \triangle WXY \cong \triangle AKL \). List the congruent corresponding parts. Use these letters for each angle.
   - \( \angle W \cong \angle A \), \( \angle X \cong \angle K \), \( \angle Y \cong \angle L \).
   - If \( \triangle WXY \cong \triangle AKL \), then the corresponding sides are equal.
   - \( WX = AL \), \( XY = KL \), \( WY = AK \).
Lesson 4-3

Triangle Congruence by ASA and AAS

Lesson Objective

Prove two triangles congruent using the ASA Postulate and the AAS Theorem.

Vocabulary

- ASA (Angle-Side-Angle)
- AAS (Angle-Angle-Side)
- ASA Postulate
- AAS Theorem

Example

Using ASA

Suppose that \( \angle A \cong \angle C \) and \( \angle A \cong \angle C \). Name the triangles that are congruent by the ASA Postulate.

The diagram shows \( \triangle ABC \) and \( \triangle DEF \). Use \( \angle A \cong \angle C \) and \( \angle A \cong \angle C \) to prove that \( \triangle ABC \cong \triangle DEF \).

Quick Check

1. Using only the information in the diagram, can you conclude that \( \triangle FNE \cong \triangle DEF \)? Explain.

No; Only one angle and one side are shown to be congruent. At least one more congruent side or angle is necessary to prove congruence with SAS, ASA, or AAS.

Lesson 4-4

Using Congruent Triangles: CPCTC

Lesson Objective

Use triangle congruence and CPCTC to prove that parts of two triangles are congruent.

Vocabulary

- CPCTC (Corresponding Parts of Congruent Triangles)
- Congruent

Example

Congruence Statements

The diagram shows the frame of an umbrella. What congruence statement(s) can you write from the diagram, in which \( \angle E \cong \angle G \) and \( \angle C \cong \angle H \)?

The congruence statements that remain to be proved are \( \triangle ABC \cong \triangle EFG \) and \( \angle C \cong \angle H \).

Quick Check

1. In Example 3, what can you say about \( \angle A \) and \( \angle B \)? Explain.

They are congruent, because supplements of congruent angles are congruent.

2. Recall Example 3. About how wide was the river if the officer paced off 20 paces and each pace was about 2 feet long?

50 feet
Lesson 4-5

Isosceles and Equilateral Triangles

Lesson Objective

- Use and apply properties of isosceles triangles

Vocabulary and Key Concepts

1. **Isosceles Triangle**: A triangle with at least two sides of equal length.
   - **Legs**: The two congruent sides of an isosceles triangle.
   - **Base**: The side opposite the vertex angle.
   - **Vertex Angle**: The angle formed by the two congruent sides.

2. **Equilateral Triangle**: A triangle with all three sides of equal length.
   - **Angles**: Each angle measures 60 degrees.

Theorem 4-3: Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

**Example**

The diagram shows that \( \triangle ABC \) is isosceles. Since \( \overline{AB} \) and \( \overline{CB} \) are congruent, the angles opposite \( \overline{AB} \) and \( \overline{CB} \) are congruent.

Quick Check

1. In the figure, \( \triangle ABC \) is isosceles. If \( \overline{AB} \) and \( \overline{CB} \) are congruent, then the angles opposite \( \overline{AB} \) and \( \overline{CB} \) are congruent.

Lesson 4-6

Congruence in Right Triangles

Lesson Objective

- Prove triangles congruent using the HL Theorem

Vocabulary and Key Concepts

1. **Right Triangle**: A triangle with one right angle.
   - **Legs**: The two sides that form the right angle.
   - **Hypotenuse**: The side opposite the right angle.

2. **Hypotenuse-Leg (HL) Theorem**: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

**Example**

Using the Hypotenuse-Leg Theorem

Given: \( \triangle ABC \) and \( \triangle DCE \) are right triangles. \( \overline{BC} \cong \overline{CE} \) and \( \overline{BD} \cong \overline{CE} \). Prove: \( \triangle ABC \cong \triangle DCE \)

**Statements**

- \( \overline{BC} \cong \overline{CE} \)
- \( \overline{BD} \cong \overline{CE} \)
- \( \angle BCA \cong \angle ECD \)
- \( \triangle ABC \cong \triangle DCE \)

**Reasons**

1. Given
2. Definition of Right Triangle
3. \( \overline{BC} \cong \overline{CE} \)
4. \( \overline{BD} \cong \overline{CE} \)
5. Hypotenuse-Leg Theorem

Quick Check

1. Which two triangles are congruent by the HL Theorem? Write a correct congruence statement.

\( \triangle ABC \cong \triangle DCE \)

2. You know that two legs of one right triangle are congruent to two legs of another right triangle. Explain how to prove the triangles are congruent.

Since all right angles are congruent, the triangles are congruent by SAS.
Lesson 5-1  Midsegments of Triangles

**Lesson Objective**
- Use properties of midsegments to solve problems

**Vocabulary and Key Concepts**

<table>
<thead>
<tr>
<th>Theorem 5-1: Triangle Midsegment Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side, and its length is 1/2 the length of the third side.</td>
</tr>
</tbody>
</table>

**Examples**

1. **Finding Lengths**: In \( \triangle DEF \), \( M \) and \( N \) are midpoints.
   - The perimeter of \( \triangle MNP \) is 60. Find \( NP \).
   - Because the perimeter of \( \triangle MNP \) is 60, you can find \( NP \).
   - \( NP = MN + MP = 60 \) \( \text{Definition of perimeter} \)
   - Simplify and solve for \( NP \).
   - \( NP = 30 \), \( MN = 15 \), \( MP = 15 \).
   - Use the Triangle Midsegment Theorem to find \( YZ \).
   - \( MP = YZ \) \( \text{Triangle Midsegment Theorem} \)
   - \( MP = 15 \) \( \text{Substitute} \)
   - \( YZ = MP \) \( \text{Substitute} \)
   - Multiply each side by 2.

**Quick Check**

1. **Proving Two Other Triangles Congruent**
   - \( m\angle HAB = 2m\angle HBC = 2m\angle HAC \)
   - Prove: \( \triangle HAB \cong \triangle HBC \)
   - Given: \( m\angle HAB = 2m\angle HBC \)
   - \( \text{Definition of congruent angles} \)

2. **Critical Thinking**: Find \( m\angle YXZ \). Justify your answers.
   - \( \text{Vertical angles are} \cong \) and \( \text{vertical angles are} \cong \text{by substitution} \)
   - \( \text{XZ} \) and \( \text{YZ} \) are \cong \text{by} \text{substitution} \)
Lesson 5-2
Bisectors in Triangles

Vocabulary and Key Concepts

Lesson Objective
See properties of perpendicular bisectors and angle bisectors.
NAP 2005 Strand: Geometry
Topic: Relationships Among Geometric Figures
Local Standards:

Thm 5-2: Perpendicular Bisector Theorem
Theorem: If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Thm 5-3: Converse of the Perpendicular Bisector Theorem
Theorem: If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

Thm 5-4: Angle Bisector Theorem
Theorem: If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

Thm 5-5: Converse of the Angle Bisector Theorem
Theorem: If a point is equidistant from the sides of the angle, then it is on the angle bisector.

Example

Quick Check

a. According to the diagram, how far is K from \( \overline{EF} \)? From \( \overline{EF} \):

b. What can you conclude about \( \overrightarrow{LM} \)?

Find the value of \( x \):

Find \( m \angle ABD \):

Finding the Circumcenter

Finding Lengths of Medians

Lesson 5-3
Concurrent Lines, Medians, and Altitudes

Vocabulary

Concurrent lines are three or more lines that meet in one point.

The point of concurrency is the point where three or more lines intersect.

A circle circumscribes a polygon when the vertices of the polygon are on the circle.

The circumcenter of a triangle is the point of concurrency of the perpendicular bisectors of a triangle.

Examples

Finding the Circumcenter

Finding the Circumcenter

Finding the Circumcenter

Quick Check

a. Find the coordinates of the circle that circumscribes \( \triangle XYZ \).

b. Critical Thinking: In Example 3, explain why it is not necessary to find the third perpendicular bisector.

Finding the Circumcenter

Finding the Circumcenter

Finding the Circumcenter

Quick Check

a. Find the center of the circle that you can circumscribe about the triangle with vertices \((0, 0), (-8, 0), \) and \((0, 6)\):

b. Using the diagram in Example 2, find \( A'B'C' \). Check that \( WF = AB = WF' \)

Local Standards: ____________________________________________________________________

NAEP 2005 Strand: Geometry
Topics: Relationships Among Geometric Figures

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Lesson 5-4

Inequalities, Converse, and Indirect Reasoning

**Lesson Objectives**
- Write the negation of a statement.
- Use indirect reasoning.

**Key Concepts**

**Negation, Inverse, and Contrapositive Statements**

<table>
<thead>
<tr>
<th>Symbolic Form</th>
<th>You Read It</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \neg p )</td>
<td>not ( p )</td>
</tr>
<tr>
<td>( \neg q )</td>
<td>not ( q )</td>
</tr>
</tbody>
</table>

The negation of a statement is the opposite truth value from that of the original statement.

The negation of a conditional statement negates both the hypothesis and the conclusion.

A conditional and its contrapositive always have the same truth value.

**Quick Check**

1. Write the negation of each statement.
   a. \( \neg \text{Today is Tuesday} \)
   b. \( \neg \text{Today is not Tuesday} \)

2. Write (a) the inverse and (b) the contrapositive of Maria Angela's statement: "If you don't stand for something, you'll fall for anything."
   a. \( \neg \neg \text{If you stand for something, you'll fall for anything} \)
   b. \( \neg \text{If you don't stand for something, you won't fall for anything} \)

**Examples**

1. Writing the Negation of a Statement
   Write the negation of "\( \neg ABCD \) is a convex polygon.
  "
   The negation of a statement has the opposite truth value.
   The negation of an original statement is "\( \neg ABCD \) is a concave polygon."

2. Writing the Inverse and Contrapositive
   a. \( \neg \text{If } \neg \text{Today is Tuesday} \)
   b. \( \neg \text{If } \neg \text{Today is not Tuesday} \)

3. Writing the Inverse and Contrapositive Statements
   \( \neg \text{Hypothesis} \rightarrow \neg \text{Conclusion} \)
   \( \neg \text{Conclusion} \rightarrow \neg \text{Hypothesis} \)

4. Writing the inverse and contrapositive of the conditional statement "If \( \triangle ABC \) is equilateral, then it is isosceles."
   Inverse: \( \neg \text{If } \triangle ABC \text{ is equilateral, then it is isosceles} \)
   Contrapositive: \( \neg \text{If } \triangle ABC \text{ is not isosceles, then it is not equilateral} \)

5. Writing the inverse and contrapositive of the conditional statement "If \( \text{a} < \text{c} \), then \( \text{b} < \text{d} \)."
   Inverse: \( \neg \text{If } \text{a} < \text{c} \), then \( \neg \text{b} < \text{d} \)
   Contrapositive: \( \neg \text{If } \neg \text{b} < \text{d} \), then \( \neg \text{a} < \text{c} \)

6. Writing the inverse and contrapositive of the conditional statement "If the measure of \( \angle ABC \) is not more than 70, then it is not a straight angle."
   Inverse: \( \neg \text{If the measure of } \angle ABC \text{ is not more than 70, then it is not a straight angle.} \)
   Contrapositive: \( \neg \text{If the measure of } \angle ABC \text{ is more than 70, then it is not a straight angle.} \)

7. Writing the inverse and contrapositive of the conditional statement "If \( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon."
   Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
   Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

8. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
   Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
   Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

9. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
   Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
   Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

10. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
    Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
    Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

11. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
    Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
    Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

12. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
    Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
    Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

13. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
    Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
    Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

14. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
    Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
    Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

15. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
    Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
    Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

16. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
    Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
    Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)

17. Writing the inverse and contrapositive of the conditional statement "\( \angle ABC \) is not a straight angle, then \( \text{ABCD} \) is not a convex polygon.
    Inverse: \( \neg \text{If } \angle ABC \text{ is not a straight angle, then } \neg \text{ABCD is a convex polygon.} \)
    Contrapositive: \( \neg \text{If } \text{ABCD is a convex polygon, then } \angle ABC \text{ is a straight angle.} \)
Lesson 6-2 Properties of Parallelograms

Name_____________________________________ Class____________________________ Date________________

Vocabulary and Key Concepts.

Examples.

Using Algebra

1. Find the values of x and y in \( \triangle KLMN \).

2. Find the value of \( z \) in \( \triangle DEF \).
Lesson 6-3
Proving That a Quadrilateral Is a Parallelogram

Lesson Objective
- Use properties of diagonals of parallelograms and rectangles.
- Determine whether a parallelogram is a rectangle.

Key Concepts
- Theorem 6-6: If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- Theorem 6-14: If one diagonal of a parallelogram bisects two angles of the parallelogram, then the parallelogram is a rhombus.

Examples
1. Finding Values for Diagonals: Find values for $x$ and $y$ for which $ABCD$ must be a parallelogram.

- $ABCD$ is a parallelogram if $BD = BC = AD = AD$. The sum of the measures of the angles of a quadrilateral is $360°$, so the sum of the measures of the angles of a parallelogram is $180°$ for each diagonal.

2. Quick Check.

a. Find the value of $y$ for which $PQRS$ is a parallelogram.

- $PQRS$ is a parallelogram if $PS = QR$ and $QR = SR$. Because both pairs of opposite angles are congruent, the quadrilateral is a parallelogram.

b. Find the length of each diagonal.

- The diagonals of a rectangle bisect each other. The diagonals of a rhombus are perpendicular.

- Find the measures of the numbered angles in the rhombus.

- The length of each diagonal is $5$. Find the measures of the numbered angles in the rhombus.

- The measures of the numbered angles in the rhombus are complementary because the four angles formed by the diagonals must have measures $x$ and $y$. If $ABCD$ is a parallelogram, then the sum of the measures of the angles of a parallelogram is $180°$ for each diagonal.

- The length of each diagonal is $5$. Find the length of each diagonal.

- The length of each diagonal is $5$. Find the length of each diagonal.

- The measures of the numbered angles in the rhombus are supplementary because the four angles formed by the diagonals.
Lesson 6-5

Lesson Objective

- Proving congruency

Vocabulary and Key Concepts

**Trapezoids**

- Theorem 6-15: The base angles of an isosceles trapezoid are congruent.

- Theorem 6-16: The diagonals of an isosceles trapezoid are congruent.

- Kites

- Theorem 6-17: The diagonals of a kite are perpendicular.

**Examples**

1. **Finding Angle Measures in Trapezoids**

   Given: \( m \angle XYZ = 120^\circ \) and \( m \angle YXZ = 80^\circ \).

   \[ m \angle XYZ + m \angle YXZ = 180^\circ \]

   \[ 120^\circ + 80^\circ = 200^\circ \]

   Subtract \( 200^\circ \) from each side.

   \[ m \angle YXZ = 180^\circ - 200^\circ = -20^\circ \]

   Since angles cannot be negative, the angle measure is not valid. Review the given information or the construction for any possible errors.

   Conclusion: The angle measures are not congruent as stated in the problem setup. Ensure all given angle measures are correct.

Quick Check

1. In the isosceles trapezoid, \( m \angle Z = 70^\circ \). Find \( m \angle Y \). Express your answer as a fraction.

2. Find \( m \angle 1 \), \( m \angle 2 \), and \( m \angle 3 \) in the kite.

   \[ m \angle 1 = 80^\circ \]

   \[ m \angle 2 = 150^\circ \]

   \[ m \angle 3 = 48^\circ \]

Lesson 6-6

Lesson Objective

- Finding angle measures in parallelograms

Vocabulary and Key Concepts

- Parallelograms

- Diagonals of a parallelogram are congruent and bisect each other.

- Summary of parallelogram properties

Example

1. **Finding Angles in Parallelogram**

   Given: \( \overline{AB} = \overline{CD} \) and \( \overline{AD} = \overline{BC} \).

   \[ \angle A = (90^\circ + B) \]

   \[ \angle C = (90^\circ + D) \]

   Since \( \angle A = \angle C \) and \( \angle B = \angle D \) in a parallelogram, the expressions for the angles are equivalent.

Quick Check

1. Use the properties of parallelogram \( \overline{ABCD} \) to find the missing coordinates. Do not use any new variables.

2. Find \( m \angle \angle A \) and \( m \angle \angle C \) in the parallelogram.

   \[ m \angle A = 120^\circ \]

   \[ m \angle C = 120^\circ \]

   \[ m \angle A = m \angle C \] as expected in a parallelogram.

   Conclusion: The angles are congruent, verifying the properties of parallelograms.

Lesson 6-7

Lesson Objective

- Finding angle measures in triangles

Vocabulary and Key Concepts

- Triangles

- Angles in a triangle sum to \( 180^\circ \)

- Triangle Angle-Sum Theorem

Example

1. **Finding Angle Measures in Triangle**

   Given: \( \angle A = 30^\circ \) and \( \angle B = 70^\circ \).

   \[ \angle C = 180^\circ - (30^\circ + 70^\circ) \]

   \[ \angle C = 180^\circ - 100^\circ \]

   \[ \angle C = 80^\circ \]

Quick Check

1. Use the properties of triangles to find the missing coordinates. Do not use any new variables.

   \[ m \angle 1 = 60^\circ \]
Lesson 6-7

Proofs Using Coordinate Geometry

Lesson Objective

Vocabulary and Key Concepts

Theorem 6-18: Trapezoid Midsegment Theorem
(1) The midsegment of a trapezoid is parallel to the bases.
(2) The length of the midsegment of a trapezoid is half the sum of the lengths of the bases.

Example

Using Coordinate Geometry

Draw quadrilateral ABCD by connecting the midpoints of the nonparallel opposite sides of the trapezoid.

From Lesson 6-6, you know that XYZW is a parallelogram.

Quick Check

1. Solve each proportion.
   a. \( \frac{5}{3} = \frac{x}{6} \)
   b. \( \frac{2}{a} = \frac{3}{b} \)

   Cross-Product Property

   a. Simplify
   b. Solve each side by \( \frac{2}{a} = \frac{3}{b} \)

   Cross-Product Property

   Solve each proportion.
   a. \( \frac{5}{3} = \frac{x}{6} \)
   b. \( \frac{2}{a} = \frac{3}{b} \)

   Solution:

   1. A photo that is 8 in. wide and 5 in. high is enlarged to a poster that is 2 ft wide and \( \frac{10}{3} \) ft high. What is the ratio of the height of the photo to the height of the poster?

      \[ \frac{5}{\frac{10}{3}} = \frac{3}{2} \]

   2. Solve each proportion.
      a. \( \frac{5}{3} = \frac{x}{6} \)
      b. \( \frac{2}{a} = \frac{3}{b} \)

      Solution:

      a. \( \frac{5}{3} = \frac{x}{6} \)
      b. \( \frac{2}{a} = \frac{3}{b} \)
Lesson 7-2

**Similar Polygons**

**Lesson Objectives:**
- Identify similar polygons
- Apply similar polygons

**Vocabulary and Key Concepts:**

**Similar Polygons**
- Similar figures have the same shape but not necessarily the same size. Two polygons are similar if corresponding angles are congruent and corresponding sides are proportional.
- The mathematical symbol for similarity is \( \sim \).
- The similarity ratio is the ratio of the lengths of corresponding sides of similar figures.

**Example 1:** Understanding Similarity \( \triangle ABC \sim \triangle XYZ \)
- Complete each statement:
  - a. \( m_\angle B = \angle Y \) and \( m_\angle C = \angle Z \), so \( \triangle ABC \sim \triangle XYZ \)
  - b. \( \triangle ABC \sim \triangle XYZ \)
  - c. \( \angle A \) corresponds to \( \angle X \), \( \angle B \) corresponds to \( \angle Y \), \( \angle C \) corresponds to \( \angle Z \)

**Quick Check:**
1. Refer to the diagram for Example 1. Complete:
   - \( m_\angle A = \angle Y \) and \( m_\angle C = \angle Z \), so \( \triangle ABC \sim \triangle XYZ \)

**Example 2:**
- Using Similar Figures: \( \triangle ABC \sim \triangle XYZ \)
- Find the value of \( x \).
- Because \( \triangle ABC \sim \triangle XYZ \), you can write and solve a proportion:

\[
\frac{AB}{XY} = \frac{BC}{YZ}
\]

**Quick Check:**
2. Refer to the diagram for Example 2. Find \( AB \)

**Lesson 7-3**

**Proving Triangles Similar**

**Lesson Objectives:**
- Use AA, SAS, and SSS similarity statements
- Apply AA, SAS, and SSS similarity statements

**Vocabulary and Key Concepts:**

**Postulate 7-1: Angle-Angle Similarity (AA~ Postulate)**
- If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

**Theorem 7-1: Side-Angle-Side Similarity (SAS~) Theorem**
- If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar.

**Theorem 7-2: Side-Side-Side Similarity (SSS~) Theorem**
- If the corresponding sides of two triangles are proportional, then the triangles are similar.

**Examples:**

1. Using the AA~ Postulate \( \triangle ABC \sim \triangle XYZ \)
   - Write why the triangles are similar.
   - Because \( \angle A = \angle X \) and \( \angle B = \angle Y \), \( \triangle ABC \sim \triangle XYZ \)

2. Solve for \( x \) and \( y \).
   - Direct measurement is a way of measuring things that are difficult to measure directly.

**Quick Check:**
1. In Example 1, you have enough information to write a similarity statement.
   - Do you have enough information to find the similarity ratio? Explain.

2. Explain why the triangles at the right must be similar.
   - Write a similarity statement.
   - Verify that the triangles are similar by the SSS~ Theorem; \( \triangle ABC \sim \triangle XYZ \).
Lesson 7-4

Similarity in Right Triangles

**Lesson Objective:**
- Find and use relationships in similar right triangles.

**Vocabulary and Key Concepts:**

- **Theorem 7-3:**
  - The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and similar to each other.

- **Corollary 1 to Theorem 7-3:**
  - The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

- **Corollary 2 to Theorem 7-3:**
  - The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the hypotenuse segments.

- **The geometric mean of two positive numbers a and b is the positive number x such that x^2 = ab.**

**Examples:**

1. **Finding the Geometric Mean:**
   - Find the geometric mean of 3 and 12.
   - Write a proportion.
   - Cross-Product Property.
   - Find the positive square root.
   - The geometric mean of 3 and 12 is 6.

2. **Finding Distance:**
   - At a golf course, Maria drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find x and y, their remaining distances from the cup.
   - Use Corollary 2 of Theorem 7-3 to solve for x.
   - Write a proportion.
   - Cross-Product Property.
   - Simplify.
   - Solving for x...
   - Using the Triangle-Angle-Bisector Theorem...
   - Recall Example 2: Find the distance between Maria’s ball and Gabriel’s ball.

**Quick Check:**

1. Find the geometric mean of 5 and 20.
2. Find the value of x.

---

Lesson 7-5

Proportions in Triangles

**Lesson Objectives:**
- Use the Side-Splitter Theorem.
- Use the Triangle-Angle-Bisector Theorem.

**Key Concepts:**

- **Theorem 7-4:**
  - If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

- **Theorem 7-5:**
  - **Triangle-Angle-Bisector Theorem:**
    - If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

**Corollary to Theorem 7-4:**

- If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

**Examples:**

1. **Using the Side-Splitter Theorem:**
   - Find y.
   - Write a proportion.
   - Cross-Product Property.
   - Solve for y.

2. **Using the Triangle-Angle-Bisector Theorem:**
   - Find the value of x.
   - Use Corollary 2 of Theorem 7-3 to solve for x.
   - Write a proportion.
   - Cross-Product Property.
   - Distributive Property.
   - Solve for x.

**Quick Check:**

1. Use Corollary 2 of Theorem 7-3 to solve for y.
2. Use the Side-Splitter Theorem to find the value of x.
Lesson 8-1: The Pythagorean Theorem and Its Converse

**Vocabulary and Key Concepts**

**Theorem 8-5: Pythagorean Theorem**

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

\[ a^2 + b^2 = c^2 \]

**Theorem 8-6: Converse of the Pythagorean Theorem**

If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

A Pythagorean triple is a set of three positive integers that satisfy the equation \( a^2 + b^2 = c^2 \).

**Examples**

1. **Pythagorean Triples**
   - Find the length of the hypotenuse of \( \triangle ABC \).
   - The lengths of the sides of \( \triangle ABC \) form a Pythagorean triple because they are positive integers that satisfy \( a^2 + b^2 = c^2 \).

2. **Finding the Length of the Hypotenuse**
   - Find the length of the hypotenuse of a 45°-45°-90° triangle with legs of length 5.
   - Use the properties of a 45°-45°-90° triangle.
   - \( \text{hypotenuse} = \sqrt{2} \times \text{leg} \)

3. **Using the Length of One Side**
   - Find the length of the hypotenuse of a 30°-60°-90° triangle with shorter leg of length 5.
   - Use the properties of a 30°-60°-90° triangle.
   - \( \text{hypotenuse} = 2 \times \text{shorter leg} \)

Quick Check

1. **Is it a Right Triangle?**
   - In this triangle a right triangle?
   - Because \( a^2 + b^2 = c^2 \), this is a right triangle.

2. **Quick Check**
   - A 45°-45°-90° triangle has legs of length 16 and 16. Is the triangle a right triangle?
   - Yes, the length of the hypotenuse is 16.

Local Standards: ____________________________________

**Lesson Objectives**

- Use the Pythagorean Theorem
- Use the Converse of the Pythagorean Theorem

**Topic:** Relationships Among Geometric Figures

**Local Standards:**

- [ ]

---

Lesson 8-2: Special Right Triangles

**Key Concepts**

**Theorem 8-5: 45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, both legs are congruent, and the length of the hypotenuse is \( \sqrt{2} \) times the length of a leg.

\[ a = b \quad \text{and} \quad c = \sqrt{2} \times a \]

**Theorem 8-4: 30°-60°-90° Triangle Theorem**

In a 30°-60°-90° triangle, the hypotenuse is twice the length of the shorter leg.

\[ c = 2 \times a \]

**Examples**

1. **Finding the Length of the Hypotenuse**
   - Find the length of the hypotenuse of the 45°-45°-90° triangle.
   - \( \text{hypotenuse} = \sqrt{2} \times \text{leg} \)

2. **Using the Length of One Side**
   - Find the length of the hypotenuse of a 30°-60°-90° triangle with shorter leg of length 6.
   - \( \text{hypotenuse} = 2 \times \text{shorter leg} \)

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Name: __________________ Class: __________________ Date: ________________

Name: __________________ Class: __________________ Date: ________________

Name: __________________ Class: __________________ Date: ________________

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Lesson 8-3

The Tangent Ratio

**Vocabulary**
- The ratio of the length of the leg opposite \( \angle C \) to the length of the leg adjacent to \( \angle C \).
- \( \tan \) means tangent of \( \angle C \).

**Examples**

1. **Writing Tangent Ratios** Write the tangent ratios for \( \angle B \) and \( \angle t \).

   \[
   \tan B = \frac{opposite}{adjacent} = \frac{BC}{AC} \\
   \tan t = \frac{opposite}{adjacent} = \frac{AC}{BC}
   \]

   You can abbreviate the equation as \( \tan \angle B = \frac{BC}{AC} \) and \( \tan \angle t = \frac{AC}{BC} \).

Lesson 8-4

Sine and Cosine Ratios

**Vocabulary**
- The sine of \( \angle A \) is the ratio of the length of the leg opposite \( \angle A \) to the length of the hypotenuse.
- The cosine of \( \angle A \) is the ratio of the length of the leg adjacent to \( \angle A \) to the hypotenuse.

**Examples**

1. **Writing Sine and Cosine Ratios** Use the triangle to find \( \sin \angle T \), \( \cos \angle T \), \( \sin \angle G \), and \( \cos \angle G \). Write your answers in simplest terms.

   \[
   \sin T = \frac{opposite}{hypotenuse} = \frac{25}{26} \\
   \cos T = \frac{adjacent}{hypotenuse} = \frac{12}{26} \\
   \sin G = \frac{opposite}{hypotenuse} = \frac{20}{26} \\
   \cos G = \frac{adjacent}{hypotenuse} = \frac{15}{26}
   \]

Quick Check

1. Write the sine and cosine ratios for \( \angle X \) and \( \angle Y \):

   \[
   \sin X = \frac{opposite}{hypotenuse} = \frac{20}{28} \\
   \cos X = \frac{adjacent}{hypotenuse} = \frac{24}{28} \\
   \sin Y = \frac{opposite}{hypotenuse} = \frac{40}{28} \\
   \cos Y = \frac{adjacent}{hypotenuse} = \frac{16}{28}
   \]

2. In Example 2, suppose that the angle the wire makes with the ground is 50°. What is the height of the flagpole to the nearest foot?

   The flagpole, wire, and ground form a right triangle with the wire as the hypotenuse. Because you know an angle and the measure of its hypotenuse, you can use the cosine ratio to find the height of the flagpole.

   \[
   \cos 50° = \frac{height}{hypotenuse} \\
   \text{For height:} \\
   20 \text{ ft} = \frac{\text{height}}{28} \\
   \text{Solve for height:} \\
   \text{The flagpole is about 15 feet tall.}
   \]
Lesson 8-5: Angles of Elevation and Depression

### Lesson Objectives

- Describe angles of elevation and depression to solve problems.
- Identify angles of elevation and depression as they relate to the situation shown.
- Determine angles of elevation and depression from given information.

### Vocabulary

- **Angle of Elevation**: The angle formed by a horizontal line and the line of sight to an object above the horizontal line.
- **Angle of Depression**: The angle formed by a horizontal line and the line of sight to an object below the horizontal line.

### Examples

#### 1. Identifying Angles of Elevation and Depression

Describe $\angle 1$ and $\angle 2$ as they relate to the situation shown.

- **Solution**:
  - $\angle 1$ is the angle of elevation from the airplane to the building.
  - $\angle 2$ is the angle of depression from the building to the airplane.

#### 2. Identifying Angles of Elevation and Depression

Describe each angle as it relates to the situation in Example 1.

- **Solution**:
  - $\angle 1$: The angle of elevation from the airplane to the building.
  - $\angle 2$: The angle of depression from the building to the airplane.

### Quick Check

1. Describe each angle as it relates to the situation in Example 1.

   - **Solution**:
     - $\angle a$: The angle of depression from the building to the person on the ground.
     - $\angle b$: The angle of elevation from the person on the ground to the building.

2. An airplane pilot sees a life raft at a $26^\circ$ angle of depression. The airplane’s altitude is $3$ km. What is the airplane’s horizontal distance from the raft?

   - **Solution**:
     - Use sine and cosine to find the horizontal distance.
     - $x = 3 \tan 26^\circ 
     - $x \approx 2.1$ km

### Lesson 8-6: Vectors

#### Lesson Objectives

- Describe vectors to solve problems that involve vector quantities.
- Use the sine and cosine ratios to find the values of variables.

#### Vocabulary and Key Concepts

- **Vector**: Any quantity with magnitude (size) and direction.
- **Resultant Vector**: The sum of two vectors.

#### Adding Vectors

For $\mathbf{u} = (2, 3)$ and $\mathbf{v} = (1, 2)$, find $\mathbf{u} + \mathbf{v}$.

- **Solution**:
  - Add the coordinates:
    - $x = 2 + 1 = 3$
    - $y = 3 + 2 = 5$
  - Resultant vector $\mathbf{w} = (3, 5)$

#### Quick Check

1. Describe the vector at the right as an ordered pair. Give the coordinates to the nearest tenth.

   - **Solution**:
     - $x = \cos 35^\circ 
     - $x \approx 0.819 
     - $y = \sin 35^\circ 
     - $y \approx 0.574 
     - Resultant vector $\mathbf{w} = (0.8, 0.6)$
Lesson 9-1 Translations

**Lesson Objectives**
- Identify isometries
- Find translation images of figures

**Vocabulary**
- A translation (slide) is a transformation in which the preimage and the image are congruent.
- A translation (slide) is a transformation such that if a point $Q$ is reflected across line $r$, then its image $Q'$ is reflected across line $r$ and also forms congruent angles with line $r$.
- The image appears to be the same as the preimage, but appears to be an isometry.

**Examples**

1. **Identifying Isometries**
   - Does the transformation appear to be an isometry?
   - The image appears to be the same as the preimage, but appears to be an isometry.

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Lesson 9-2 Reflections

**Lesson Objectives**
- Identify reflection images of figures
- Name: ________________________ Class: __________________ Date: ________________

**Vocabulary**
- A reflection in line $x$ is a transformation such that if a point $A$ is on line $x$ then the image of $A$ is $A'$ and if a point $A$ is not on line $x$, then its image $A'$ is the perpendicular bisector of $AA'$.

**Examples**

1. **Finding Reflection Images**
   - If point $Q(1, 2)$ is reflected across line $y = 1$, what are the coordinates of its reflection image?
   - If point $Q$ is reflected across line $y = 1$, then its reflection image is the point $Q'(x, y)$ where $y = 1$.

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Quick Check

1. What are the coordinates of the image of $Q$ if the reflection line is $y = 1$?
   - $(x, y) = (1, 2)$

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Lesson 9-3 Daily Notetaking Guide

Geometry: All-In-One Answers Version B (continued)
Lesson 9-3

Vocabulary

A rotation of \(c\) about a point \(R\) is a transformation for which the following are true:

- The image of \(B\) is \(B'\), which is \(\frac{1}{2}\) of the rotation.
- For any point \(A\), \(\overrightarrow{AR} = \overrightarrow{A'R'}\), and \(\overrightarrow{OA} = \overrightarrow{OA'}\).

Examples

1. Use a protractor to draw a 60° angle at vertex \(C\) with one side \(\overline{CO}\).
2. Locate \(L\) and \(R\) in a similar manner. Then draw \(\triangle LOR\).
3. Draw the image of \(\triangle LOR\) under a 60° rotation about \(C\).

Lesson Objective

Identify the type of symmetry in a figure.

Quick Check

1. a. Draw a rectangle and all of its lines of symmetry.
   b. Name the image of \(\triangle ABC\) under a 60° rotation about \(R\).
2. Regular pentagon \(PENTA\) is divided into 5 congruent triangles. Name the image of \(\triangle PTN\) for a 60° rotation about \(M\).
3. Label the vertices of the image of \(\triangle ABC\) for a 60° rotation about \(X\).

Lesson 9-4

Vocabulary

A figure has symmetry if there is an isometry that maps the figure onto itself.

A figure has reflectional symmetry if there is symmetry that maps the figure onto itself.

Line symmetry is the same as reflectional symmetry.

The heart-shaped figure has reflectional symmetry.

The figure is its own image after one half-turn, so it has rotational symmetry.

The letter \(V\) does not have rotational symmetry because it must be rotated \(180°\) to be its own image.

Quick Check

1. Draw a rectangle and all of its lines of symmetry.
2. Angle: \(180°\)
3. Identify the type of symmetry in the regular pentagon.

Lesson Objective

Identify the type of symmetry in a figure.

Quick Check

1. Name the vertices of the image of \(\triangle ABC\) for a 60° rotation about \(X\).
2. Angle: \(180°\)
Lesson 9-5: Dilations

Lesson Objective
Locate dilation images of figures.

Vocabulary
A dilation is a transformation with center C and scale factor r for which the following are true:

1. The image of C is (that is, C' = C).
2. For any point R, R' is on CR and CR' = rCR.

Example
A dilation is a transformation with center (0, 0) and scale factor 1/2.

The dilation is a reduction with center (0, 0) and scale factor 1/2.

Vocabulary and Key Concepts
A dilation is a transformation with center C and scale factor r for which the following are true:

1. The image of C is (that is, C' = C).
2. For any point R, R' is on CR and CR' = rCR.

Example
A dilation is a transformation with center (0, 0) and scale factor 1/2.

The dilation is a reduction with center (0, 0) and scale factor 1/2.

Lesson 9-6: Compositions of Reflections

Lesson Objective
Use a composition of reflections.

Vocabulary and Key Concepts
Theorem 9-1: A translation or rotation is a composition of two reflections.

Theorem 9-2: A composition of reflections across two parallel lines is a translation.

Theorem 9-3: A composition of reflections across two intersecting lines is a rotation.

Theorem 9-4: Fundamental Theorem of Isometries
In a plane, one of two congruent figures can be mapped onto the other by a composition of at most three reflections.

Theorem 9-5: Isometry Classification Theorem
There are only four isometries. They are the following:

- Reflection
- Translation
- Rotation
- Glide reflection

A glide reflection is the composition of a glide translation and a reflection across a line parallel to the direction of translation.

Example
The composition of two reflections across intersecting lines is a rotation.

The center of rotation is the point where the lines intersect, and the angle between the lines is the angle formed by the intersecting lines. In the composition of two reflections, the center of rotation is the point where the lines intersect.

Quick Check
1. Quadrilateral JKLM is a dilation image of quadrilateral ABCD. Describe the dilation.

2. The height of a tractor-trailer truck is 4.2 m. The scale factor for a model truck is 1/200. Find the height of the model to the nearest centimeter.

Scale Drawings
The scale factor on a museum's floor plan is 1:200. The museum wing measures by . Describe the dilation.

Quick Check
1. Quadrilateral JKLM is a dilation image of quadrilateral ABCD. Describe the dilation.

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The scale factor on a museum's floor plan is 1:200. The museum wing measures by . Describe the dilation.

Quick Check
1. Quadrilateral JKLM is a dilation image of quadrilateral ABCD. Describe the dilation.

2. The height of a tractor-trailer truck is 4.2 m. The scale factor for a model truck is 1/200. Find the height of the model to the nearest centimeter.
Lesson 9-7

**Tessellations**

**Lesson Objectives**
- Identify transformation in tessellations, and figures that will tessellate.
- Identify symmetry in tessellations.

**Vocabulary and Key Concepts**

**Theorem 9-6**
Every triangle tessellates.

**Theorem 9-7**
Every quadrilateral tessellates.

A tessellation is a repeating pattern of figures that completely covers a plane.

**Translational Symmetry**
Maps a figure onto itself.

**Glide Reflectional Symmetry**
Maps a figure onto itself.

**Examples**

**Identifying the Transformation in a Tessellation** Identify the repeating figures and a transformation in the tessellation.

1. A repeated combination of an **octagon** and one adjoining **square** will completely cover the plane without gaps or overlaps. Use arrows to show a translation.

2. List the symmetries in the tessellation.

   Starting at any vertex, the tessellation can be mapped onto itself using **translational** symmetry; as can be seen by sliding any triangle onto a copy of itself along any of the lines.

   **Quick Check.**
   1. Identify a transformation and outline the smallest repeating figure in the tessellation below.

   **Identifying Symmetries in Tessellations** List the symmetries in the tessellation.

   Starting at any vertex, the tessellation can be mapped onto itself using **translational** symmetry. The tessellation can be mapped onto itself using **reflectional** symmetry, as can be seen by sliding any triangle onto a copy of itself along any of the lines.

   **Quick Check.**
   1. Identify a transformation and outline the smallest repeating figure in the tessellation below.

   **Exercise:**
   1. Find the area of the triangle.

   **Finding the Area of a Triangle**

   The area of a triangle is the product of half the length of the base and the corresponding altitude.

   \[ A = \frac{1}{2} \times \text{base} \times \text{height} \]

   **Example:**
   A triangle has a base of 10 cm and a height of 5 cm. Find the area of the triangle.

   \[ A = \frac{1}{2} \times 10 \times 5 = 25 \text{ cm}^2 \]

   **Quick Check.**
   1. Find the area of the triangle.

   **Finding the Area of a Parallelogram**

   The area of a parallelogram is the product of the length of the base and the corresponding altitude.

   \[ A = \text{base} \times \text{height} \]

   **Example:**
   A parallelogram has a base of 10 cm and a height of 5 cm. Find the area of the parallelogram.

   \[ A = 10 \times 5 = 50 \text{ cm}^2 \]

   **Quick Check.**
   1. Find the area of the parallelogram.

   2. Find the area of the triangle.
Lesson 10-2
Areas of Trapezoids, Rhombuses, and Kites

Lesson Objectives
- Find the area of a trapezoid
- Find the area of a rhombus or a kite

Vocabulary and Key Concepts

Theorem 10-4: Area of a Trapezoid
The area of a trapezoid is half the product of the height and the sum of the bases.

\[ A = \frac{1}{2} (b_1 + b_2) \]

Theorem 10-5: Area of a Rhombus or a Kite
The area of a rhombus or a kite is half the product of the lengths of its diagonals.

\[ A = \frac{1}{2} d_1 d_2 \]

Examples

1. Applying the Area of a Trapezoid
A car window is shaped like the trapezoid shown. Find the area of the window.

\[ A = \frac{1}{2} (8 + 30) \times 20 \]
\[ A = \frac{1}{2} \times 38 \times 20 \]
\[ A = 380 \text{ ft}^2 \]

2. Finding the Area of a Rhombus
Find the area of rhombus RSTU.

\[ A = \frac{1}{2} d_1 d_2 \]
\[ A = \frac{1}{2} (12 \text{ ft})(5 \text{ ft}) \]
\[ A = 30 \text{ ft}^2 \]

Quick Check

1. Find the area of a trapezoid with bases 11 cm and 13 cm and a height of 6 cm.
2. Critical Thinking
In Example 2, explain how you can use a Pythagorean triple to conclude that \( d_1 = 5 \text{ ft} \).

Lesson 10-3
Areas of Regular Polygons

Lesson Objectives
- Find the area of a regular polygon

Vocabulary and Key Concepts

Theorem 10-6: Area of a Regular Polygon
The area of a regular polygon is half the product of the apothem and the perimeter.

\[ A = \frac{1}{2} aP \]

Examples

1. Finding Angle Measures
This regular heptagon has an apothem and radius drawn. Find the measures of each numbered angle.

\[ \text{Angle} = \frac{360}{n} \]

2. Finding the Area of a Regular Polygon
A heptagon is in the shape of a regular octagon. Each side is 10.8 ft. Find the area of the heptagon.

\[ A = \frac{1}{2} aP \]

Quick Check

1. At the right, a portion of a regular octagon has an apothem and a radius drawn. Find the measures of each numbered angle.

\[ \text{Angle} = \frac{360}{n} \]

2. Find the area of a regular pentagon with 12.6-cm sides and an 8-cm apothem.
Lesson 10-4
Perimeters and Areas of Similar Figures

Key Concepts

- Finding Ratios in Similar Figures: The triangles at the right are similar. Find the ratio of their perimeters and of their areas.
- Finding Areas Using Similar Figures: The ratio of the lengths of the corresponding sides of the regular octagons is 5 : 4. The area of the larger octagon is 320 ft². Find the area of the smaller octagon.

Examples

1. The triangles at the right are similar. Find the ratio of their perimeters and of their areas.

2. Finding Ratios in Similar Figures: The triangles at the right are similar. Find the ratio of their perimeters and of their areas.

3. Finding Areas Using Similar Figures: The ratio of the lengths of the corresponding sides of the regular octagons is 5 : 4. The area of the larger octagon is 320 ft². Find the area of the smaller octagon.

4. The triangles at the right are similar. Find the ratio of their perimeters and of their areas.

5. Finding Ratios in Similar Figures: The triangles at the right are similar. Find the ratio of their perimeters and of their areas.

6. Finding Areas Using Similar Figures: The ratio of the lengths of the corresponding sides of the regular octagons is 5 : 4. The area of the larger octagon is 320 ft². Find the area of the smaller octagon.

Quick Check

1. Two similar polygons have corresponding sides in the ratio 5 : 7. a. Find the ratio of their perimeters. b. Find the ratio of their areas.

2. The corresponding sides of two similar parallelograms are in the ratio 3 : 7. The area of the smaller parallelogram is 56 in². Find the area of the larger parallelogram.

3. The similarity ratio of the dimensions of two similar pieces of window glass is 3 : 5. The smaller piece costs $2.50. What should be the cost of the larger piece?
Lesson 10-6 Circles and Arcs

Vocabulary and Key Concepts

Postulate 10-1: Arc Addition Postulate
The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Theorem 10-9: Circumference of a Circle
The circumference of a circle is 

\[ C = 2\pi r \]

A circle is a set of all points equidistant from a given point called the center.

A radius is a segment that has one endpoint at the center and the other endpoint on the circle.

A central angle is an angle whose vertex is at the center of the circle.

Circumference of a circle is the distance around the circle.

Theorem 10-10: Arc Length
The length of an arc of a circle is the product of the ratio of the measure of the arc to 360° and the circumference of the circle.

\[ \text{Length of arc} = \frac{\text{Measure of the arc}}{360} \times 2\pi r \]

Examples:

1. Find \( m\text{ADB} \) if \( m\text{ABC} = 150° \).

2. Find the length of a semicircle with radius 1.3 m in terms of \( \pi \).

Quick Check:
1. Use the diagram in Example 1. Find \( m\text{YCD}, m\text{YDP}, m\text{YPF}, \) and \( m\text{YHP} \).

2. Find the length of a semicircle with radius 1.3 in in terms of \( \pi \).

Lesson 10-7 Areas of Circles and Sectors

Vocabulary and Key Concepts

Theorem 10-11: Area of a Circle
The area of a circle is the product of \( \pi \) and the square of the radius.

\[ A = \pi r^2 \]

Theorem 10-12: Area of a Sector of a Circle
The area of a sector of a circle is the product of the ratio of the measure of the arc to 360° and the area of the circle.

\[ \text{Area of sector} = \frac{\text{Measure of the arc}}{360} \times \pi r^2 \]

Examples:

1. Finding the Measures of Arcs

Find \( m\text{XY} \) and \( m\text{DY} \) in circle C.

2. Finding the Area of a Sector of a Circle

Find the area of sector \( \text{ACB} \) if \( \text{arc } AB \) is 12 in. and \( \text{arc } BC \) is 10 in.

Quick Check:
1. How much more pizza is in a 14-in.-diameter pizza than in a 12-in. pizza?

2. Critical Thinking: A circle has a diameter of 20 cm. What is the area of a sector bounded by a 36° major arc? Round your answer to the nearest tenth.

\[ 18.8 \text{ cm}^2 \]
Example

Finding Probability Using Segments

A gnat lands at random on the edge of the ruler below. Find the probability that the gnat lands on a point between 2 and 10.

The length of the segment between 2 and 10 is 8.

The length of the ruler is 20.

The probability that a gnat landing randomly on the edge of the ruler below is between 2 and 10.

\[ P(2 \text{ and } 10) = \frac{8}{20} = 0.4 \]
Lesson 11-3 Surface Areas of Pyramids and Cones

Vocabulary and Key Concepts

Thm 11-2: Lateral and Surface Area of a Regular Pyramid

The lateral area of a regular pyramid is the sum of the areas of the congruent lateral faces.

The surface area of a regular pyramid is the sum of the lateral area and the area of the base.

Example

Finding Surface Area of a Pyramid

Find the surface area of a square pyramid with base edges 7.5 ft and slant height 12 ft.

L.A. of a regular pyramid is half the product of the perimeter of the base and the slant height.

S.A. = L.A. + B

Substitute.

Use the formula for lateral area of a pyramid.

Substitute.

Simplify.

Use the formula for surface area of a pyramid.

Simplify.

The surface area of the square pyramid is $106.25 \text{ ft}^2$.

Quick Check

a. Find the surface area of a cylinder with height 10 cm and radius 10 cm in terms of $\pi$.

b. The company in the Example wants to make a label to cover the cylindrical container. The label will cover the container all the way around, but will not cover any part of the top or bottom. What is the area of the label to the nearest tenth of a square inch?

Find the surface area of a pyramid: $120 \text{ ft}^2$.
Lesson 11-5: Volumes of Pyramids and Cones

Lesson Objectives
- Find the volume of a pyramid
- Find the volume of a cone

Key Concepts

1. Finding Volume of a Pyramid

The formula for the volume of a pyramid is \( V = \frac{1}{3}Bh \), where \( B \) is the area of the base and \( h \) is the height of the pyramid.

Example:

Find the volume of a square pyramid with base edges 24 m and slant height 13 m.

- The area of the base is \( 24 \times 24 = 576 \text{ m}^2 \).
- The volume of the pyramid is \( \frac{1}{3} \times 576 \times 13 = 2496 \text{ m}^3 \).

2. Finding Volume of a Cone

The formula for the volume of a cone is \( V = \frac{1}{3} \pi r^2 h \), where \( r \) is the radius of the base and \( h \) is the height of the cone.

Example:

Find the volume of a cone with radius 6 in. and height 8 in.

- The area of the base is \( \pi \times 6^2 = 36 \pi \text{ in}^2 \).
- The volume of the cone is \( \frac{1}{3} \times 36 \pi \times 8 = 96 \pi \text{ in}^3 \).

Quick Check

1. Find the volume of a triangular prism with base edges 15 cm and height 22 cm.

- The area of the base is \( 15 \times 15 = 225 \text{ cm}^2 \).
- The volume of the prism is \( 225 \times 22 = 4950 \text{ cm}^3 \).

2. Find the volume of an oblique cone with base edge 20 m and slant height 15 m.

- The volume of the cone is \( \frac{1}{3} \pi \times 20^2 \times 15 = 2000 \pi \text{ m}^3 \).

3. A small child's tepee is in the shape of a cone with diameter 4 feet and height 3 feet. Find the volume of the tepee to the nearest cubic foot.

- The radius of the base is 2 feet.
- The area of the base is \( \pi \times 2^2 = 4 \pi \text{ ft}^2 \).
- The volume of the cone is \( \frac{1}{3} \times 4 \pi \times 3 = 4 \pi \text{ ft}^3 \).
Lesson 11-7 Areas and Volumes of Similar Solids

Lesson Objective
Vocabulary and Key Concepts

Theorem 11-10: Surface Area of a Sphere
The surface area of a sphere is four times the product of its radius and the circumference of its great circle.

Theorems and Concepts

1. If two similar solids are , then
   a. the ratio of corresponding areas is 
   b. the ratio of corresponding volumes is 

2. Given two similar solids with surface areas of 98 ft
   and 616 in
   Find the similarity ratio.

3. Find the volume to the nearest whole number of a sphere with diameter 60 in.

Quick Check

Examples

1. Finding Surface Area: The circumference of a rubber ball is 13 cm. Calculate the surface area to the nearest whole number.
   Step 1: Find the radius.
   Step 2: Use the formula for surface area to find the surface area.
   Step 3: Simplify.

2. Finding Volume: Find the volume of the sphere.
   Step 1: Use the formula for volume.
   Step 2: Substitute and simplify.

Local Standards:

NAEP 2005 Strand:
Measurement

Local Standards:

Measuring Physical Attributes

Examples:

1. Finding Surface Area: The circumference of a rubber ball is 13 cm. Calculate the surface area to the nearest whole number.
2. Finding Volume: Find the volume of the sphere.
3. Finding the Similarity Ratio: Find the similarity ratio of two similar cylinders with surface areas of 98 ft
   and 616 in
   Use the ratio of the surface areas to find the similarity ratios.

Quick Check

1. Are the two cylinders similar? If so, give the similarity ratio.
2. Find the similarity ratio of two similar prisms with surface areas 144 m
   and 738 m

Local Standards:

NAEP 2005 Strand:
Measurement

Local Standards:

Measuring Physical Attributes

Examples:

1. Finding Surface Area: The circumference of a rubber ball is 13 cm. Calculate the surface area to the nearest whole number.
2. Finding Volume: Find the volume of the sphere.
3. Finding the Similarity Ratio: Find the similarity ratio of two similar cylinders with surface areas of 98 ft
   and 224 ft
   Use the ratio of the surface areas to find the similarity ratios.

Quick Check

1. Are the two cylinders similar? If so, give the similarity ratio.
2. Find the similarity ratio of two similar prisms with surface areas 144 m
   and 738 m

Local Standards:

NAEP 2005 Strand:
Measurement

Local Standards:

Measuring Physical Attributes
Lesson 12-1

Tangent Lines

Vocabulary and Key Concepts

**Theorem 12-1**
If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.

**Theorem 12-2**
If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

**Theorem 12-3**
The two segments tangent to a circle from a point outside the circle are congruent.

A tangent to a circle is a line, segment, or ray in the plane of the circle that intersects the circle exactly one point.

To the point of tangency
A triangle is inscribed in a circle if all vertices of the triangle lie on the circle.

A triangle is circumscribed about a circle if each side of the triangle is tangent to the circle.

Quick Check

Example

**Applying Tangent Lines**
A bell tightly around two circular pulleys, as shown. Find the distance between the centers of the pulleys. Round your answer to the nearest tenth.

Draw \( \triangle \) parallel to \( \triangle \) to form rectangle \( \triangle \). Because \( \angle \) is a right angle, and \( \angle \) is a right angle. Because the radius of \( \odot \) is \( \text{cm} \), \( \text{cm} \).

Pythagorean Theorem
Solve.

Quick Check

A bell tightly around two circular pulleys, as shown. Find the distance between the centers of the pulleys.

The distance between the centers of the pulleys is about \( \text{in.} \).
Lesson 12-3

Inscribed Angles

Vocabulary and Key Concepts

Theorem 12-9: Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

Theorem 12-10

The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

Corollaries to the Inscribed Angle Theorem

1. Two inscribed angles that intercept the same arc are congruent.
2. An angle inscribed in a semicircle is a right angle.
3. The opposite angles of a quadrilateral circumscribed in a circle are supplementary.

Example

Find the measure of an angle formed by a tangent and a chord.

Solution:

Because the tangent and chord intersect, you need to find the measure of the intercepted arc. The measure of a circle is 360°. If arc $EF = 100°$, you need to find $m_{	riangle EFG}$ in order to find $m_{	riangle DEF}$. The measure of a circle is 360°, so $m_{	riangle EFG} = 360° - 100° = 260°$.

Quick Check

Find the measure of an angle formed by a tangent and a chord.

Example

Find the values of $x$.

Solution:

Using the Inscribed Angle Theorem

Find the values of $x$.

Example

Find the values of $y$.

Solution:

Using the Inscribed Angle Theorem

Find the values of $y$.

Example

Find the values of $z$.

Solution:

Using the Inscribed Angle Theorem

Find the values of $z$.

Quick Check

In the Example, find $m_{	riangle DEF}$.

Example

Find the values of $w$.

Solution:

Using the Inscribed Angle Theorem

Find the values of $w$.
Lesson 12-5

Circles in the Coordinate Plane

Lesson Objectives
- Write an equation of a circle
- Find the center and radius of a circle

Key Concepts

Theorem 12-13
The standard form of an equation of a circle with center (h, k) and radius r is:

\[(x - h)^2 + (y - k)^2 = r^2\]

Example

Writing the Equation of a Circle
Write the standard equation of a circle with center (3, 2) and radius 4:

\[(x - 3)^2 + (y - 2)^2 = 4^2\]

Using the Center and a Point on a Circle
Write the standard equation of a circle with center (5, 8) that passes through the point (15, 13):

Find the radius:

\[r = \sqrt{(15 - 5)^2 + (13 - 8)^2} = \sqrt{100 + 25} = \sqrt{125} = 13\]

Substitute (5, 8) for (h, k) and (15, 13) for (x, y):

\[(5 - 15)^2 + (8 - 13)^2 = 13^2\]

Simplify:

\[100 + 25 = 169\]

Then the standard equation of the circle with center (5, 8) and radius 13 is:

\[(x - 5)^2 + (y - 8)^2 = 13^2\]

Quick Check

1. Write the standard equation of the circle with center (2, 3) and radius 6:

\[(x - 2)^2 + (y - 3)^2 = 6^2\]

2. Write the standard equation of the circle with center (0, 0) and radius 3:

\[x^2 + y^2 = 3^2\]

3. Find the center and radius of the circle with equation:

\[x^2 + y^2 - 4x - 6y + 9 = 0\]

4. Write the standard equation of the circle with center (2, 1) and radius 2:

\[(x - 2)^2 + (y - 1)^2 = 2^2\]

Lesson 12-6

Locus: A Set of Points

Lesson Objective
- Draw and describe a locus

Vocabulary

A locus is a set of points, all of which meet a stated condition.

Examples

1. Describing a Locus in a Plane: Draw and describe the locus of points in a plane that are 3 cm from a circle with radius 3 cm.

The locus of points in the interior of C that are 3 cm from C is the circle C.

The locus of points outside C that are 3 cm from C is the circle C.

The locus of points in a plane that are 3 cm from a point on C with radius 3 cm is point C and a circle with radius 3 cm and center C.

Quick Check

1. Draw and describe the locus. In a plane, the points 2 cm from line AB.

Two lines parallel to AB, each 2 cm from AB.

2. Draw and describe the locus of points in space that are equidistant from two parallel planes.

A plane midway between the two parallel planes, parallel to and equidistant from each.
Chapter 1

Guided Problem Solving 1-1
1. They represent three-dimensional objects on a two-dimensional surface. 2. nine 3. See the figure in 4, below.

Guided Problem Solving 1-2
1. They represent three-dimensional objects on a two-dimensional surface. 2. nine 3. See the figure in 4, below.

Practice 1-3
1. Collinear points lie on the same line. 2. Answers may vary. (Some people might note that the y-coordinate of two of the points is the same so that the third point must have the same y-coordinate to be collinear. Since it does not, the points are not collinear.) 3. horizontal 4. No. 5. No. 6. All points must have the same y-coordinate, –3. 7. No. 8. (1, –1/2)

Practice 1-4
1. true 2. false 3. false 4. false 5. JK, HG 6. any three of the following pairs: EF and FJ; EF and KG; HG and JE, HG and FK; JK and EH; JK and FG; EF and FG; EH and FK; JE and KG; EH and KG; JH and HF; JF; and KG 7. planes A and B 8. planes A and C
9. Sample: $\overline{EG}$  
10. $\overline{EF}$ and $\overline{ED}$ or $\overline{EG}$ and $\overline{ED}$  
11. $\overline{FE}$, $\overline{FD}$  
12. yes  
13. Sample:  
   \[ \text{ } \]  
14. Sample:  
   \[ \text{ } \]

Guided Problem Solving 1-4

1. Opposite rays are two collinear rays with the same endpoint. 
2. a line 
3–4. See graph in Exercise 5 answer.  
5. Answers may vary. Sample: $(0,0)$ (Answers will be coordinates $(x, y)$, where $y = \frac{3}{2}x, x < 2.$)  
6. yes  
7. $L(4, 2)$  

Practice 1-5

1. 4  
2. 12  
3. 20  
4. 6  
5. 22  
6. $-3, 4$  
7. no  
8. $-2$  
9. 11  
10. 29  
11. 29

Guided Problem Solving 1-5

1. $AD \cong DC$  
2. $AD = DC$  
4. Since $AD = DC, AC = 2(AD)$.  
5. $AC = 2(12) = 24$  
6. $y = 15$  
7. $DC = AD = 12$  
8. Answers may vary.  
9. $ED = 11, DB = 11, EB = 22$  

Practice 1-6

1. any three of the following: $\angle O, \angle MOP, \angle POM, \angle 1$  
2. $\angle AOB$  
3. $\angle EOC$  
4. $\angle DOC$  
5. 51  
6. 90  
7. 141  
8. 68  
9. $\angle ABD, \angle DBE, \angle EBF, \angle DBF, \angle FBC$  
10. $\angle ABF, \angle DBC$  
11. $\angle ABE, \angle EBC$

Guided Problem Solving 1-6

1. Angle Addition Postulate  
2. supplementary angles  
3. $m\angle RQS + m\angle TQS = 180$  
4. $(2x + 4) + (6x + 20) = 180$  
5. $x = 19.5$  
6. $m\angle RQS = 43; m\angle TQS = 137$  
7. The sum of the angle measures should be 180; $m\angle RQS + m\angle TQS = 43 + 137 = 180$.  
8a. $x = 11$  
8b. $m\angle AOB = 17$  

Practice 1-7

1.  
2.  
3.  
4.  

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Guided Problem Solving 1-7

1. \( \angle DBC \cong \angle ABC \)
2. Complementary angles
3. \( \angle CBD \)  
4. \( m\angle CBD = m\angle CBA = 41 \)
5. \( m\angle ABD = m\angle CBA + m\angle CBD = 41 + 41 = 82 \)
6. \( m\angle ABE + m\angle CBA = 90 \)
7. \( m\angle DBF = m\angle ABE = 49 \)
8. Answers may vary. Sample: The sum of the measures of the complementary angles should be 90 and the sum of the measures of the supplementary angles should be 180. 
9. \( m\angle CBD = 21, m\angle FBD = 69, m\angle CBA = 21, \) and \( m\angle EBA = 69 \)

Practice 1-8

1. Distance Formula
2. The distance \( d \) between two points \( A(x_1,y_1) \) to \( B(x_2,y_2) \) is \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
3. No; the differences are opposites but the squares of the differences are the same.
4. \( XY = \sqrt{(5 - (-6))^2 + (-2 - 9)^2} = 24.7 \)
5. To the nearest tenth, \( XY = 15.6 \) units.
6. To the nearest tenth, \( XZ = 12.0 \) units.
7. \( Z \) is closer to \( X \).
8. The results are the same; e.g., \( XY = \sqrt{(6 - 5)^2 + (9 - (-2))} = \sqrt{242} \), or about 15.6 units, as before.
9. \( YZ = \sqrt{(17 - (-6))^2 + (-3 - 9)^2} = \sqrt{673} \) to the nearest tenth, \( YZ = 25.9 \) units. To the nearest tenth, \( XY + YZ + XZ = 53.5 \) units.
Geometry: All-In-One Answers Version B (continued)

Practice 1-9
1. 792 in.²  2. 2.4 m²  3. 16π  4. 7.8π  5. 26 cm; 42 cm²
6. 29 in.; 42 in.²  7. 40 m; 99 m²  8. 26; 22  9. 30; 44
10. 156.25  11. 10,000π  12. 36  13. 26; 13

Guided Problem Solving 1-9
1. six  2. It is a two-dimensional pattern you can fold to form a three-dimensional object. 3. rectangles
4. 
```
  4  4
  8
  4  4
  6

  8
```
5. 208 in.²  6. They are equal. 7. 208 in.²  8. Answers will vary. Sample; 2(4 · 6) + 2(4 · 8) + 2(6 · 8); the results are the same, 208 in.²  9. 6(7²) = 294 in.²

1A: Graphic Organizer
1. Tools of Geometry  2. Answers may vary. Sample: patterns and inductive reasoning; measuring segments and angles; basic constructions; and the coordinate plane  3. Check students’ work.

1B: Reading Comprehension
1. Answer may vary. Sample: \( \overrightarrow{AB} \parallel \overrightarrow{CD} \), \( \overrightarrow{EF} \parallel \overrightarrow{GH} \), \( \overrightarrow{JK} \equiv \overrightarrow{LM} \), \( \overrightarrow{KL} \equiv \overrightarrow{KM} \), \( m\angle AIF + m\angle FJK = 180^\circ \), \( \angle HKM \equiv \angle KMD \mid \overrightarrow{CD} \). 2. Points \( A, M, \) and \( S \) are collinear. 3. \( \overrightarrow{AB}, \overrightarrow{HI}, \) and \( \overrightarrow{LN} \) intersect at point \( M \).

1C: Reading/Writing Math Symbols
1. Line \( BC \) is parallel to line \( MN \). 2. Line \( CD \)
3. Line segment \( GH \)  4. Ray \( AB \)  5. The length of segment \( XY \) is greater than the length of segment \( ST \).
6. \( MN = XY \)  7. \( GH = 2(KL) \)  8. \( ST \parallel UV \)
9. plane \( ABC \parallel \) plane \( XYZ \)  10. \( AB \parallel DE \)

1D: Visual Vocabulary Practice

1E: Vocabulary Check
Net: A two-dimensional pattern that you can fold to form a three-dimensional figure.
Conjecture: A conclusion reached using inductive reasoning.
Collinear points: Points that lie on the same line.
Midpoint: A point that divides a line segment into two congruent segments.
Postulate: An accepted statement of fact.

1F: Vocabulary Review Puzzle
Chapter 2

Practice 2-1
1. Sample: It is 12:00 noon on a rainy day.  2. Sample: 6
3. If you are strong, then you drink milk.  4. If a rectangle is a square, then it has four sides the same length.  5. If \( x = 26 \), then \( x - 4 = 22 \); true.  6. If \( m \) is positive, then \( m^2 \) is positive; true.  7. If lines are parallel, then their slopes are equal; true.  8. Hypothesis: If you like to shop; conclusion: Visit Pigeon Forge outlets in Tennessee.  9. If you visit Pigeon Forge outlets, then you like to shop.

Guided Problem Solving 2-1
1. Hypothesis: \( x \) is an integer divisible by 3.  2. Conclusion: \( x^2 \) is an integer divisible by 3.  3. Yes, it is true. Since 3 is a factor of \( x \), it must be a factor of \( x \cdot x = x^2 \).  4. If \( x^2 \) is an integer divisible by 3 then \( x \) is an integer divisible by 3.  5. The converse is false. Counterexamples may vary. Let \( x^2 = 3 \). Then \( x = \sqrt{3} \), which is not an integer and is not divisible by 3.  6. No. The conditional is true, so there is no such counterexample.  7. No. By definition, a general statement is false if a counterexample can be provided.  8. If \( 5x + 3 = 23 \), then \( x = 4 \). The original statement and the converse are both true.

Practice 2-2
1. Two angles have the same measure if and only if they are congruent.  2. The converse, “If \( |n| = 17 \), then \( n = 17 \),” is not true.  3. If a whole number is a multiple of 5, then its last digit is either 0 or 5. If a whole number has a last digit of 0 or 5, then it is a multiple of 5.  4. If two lines are perpendicular, then the lines form four right angles. If two lines form four right angles, then the lines are perpendicular.  5. Sample: Other vehicles, such as trucks, fit this description.  6. Sample: Baseball also fits this definition.  7. Sample: Pleasing, smooth, and rigid all are too vague.  8. yes  9. no  10. yes

Guided Problem Solving 2-2
1. A good definition is clearly understood, precise, and reversible.  2. \( \angle 3 \) and \( \angle 4 \), \( \angle 5 \) and \( \angle 6 \)  3. No.  4. They are not supplementary.  5. A linear pair has a common vertex, shares a common side, and is supplementary.  6. yes  7. yes; yes  8. linear pairs: \( \angle 1 \) and \( \angle 2 \), \( \angle 3 \) and \( \angle 4 \); not linear pairs: \( \angle 1 \) and \( \angle 3 \), \( \angle 1 \) and \( \angle 4 \), \( \angle 2 \) and \( \angle 3 \), \( \angle 2 \) and \( \angle 4 \)

Practice 2-3
1. Football practice is canceled for Monday.  2. \( \triangle DEF \) is a right triangle.  3. If two lines are not parallel, then they intersect at a point.  4. If you vacation at the beach, then you like Florida.  5. Tamika lives in Nebraska.  6. not possible  7. It is not freezing outside.  8. Shannon lives in the smallest state in the United States.

Guided Problem Solving 2-3
1. conditional; hypothesis  2. Yes  3. Beth will go.  4. Anita, Beth, Aisha, Ramon  5. No; only two students went.  6. Beth, Aisha, Ramon; no—only two went.  7. Aisha, Ramon  8. The answer is reasonable. It is not possible for another pair to go to the concert.  9. Ramon

Practice 2-4
1. \( UT = MN \)  2. \( y = 51 \)  3. \( \overline{PC} \)  4. Addition; Subtraction Property of Equality; Multiplication Property of Equality; Division Property of Equality  5. Substitution  6. Substitution  7. Symmetric Property of Congruence  8. Definition of Complementary Angles; 90; Substitution; 3x, Simplify; 3x, 84, Subtraction Property of Equality; 28, Division Property of Equality

Guided Problem Solving 2-4
1. Angle Addition Postulate  2. Substitution Property of Equality; Simplify; Addition Property of Equality; Division Property of Equality  3. 40  4. yes; yes  5. 13; 13

Practice 2-5
1. 30  2. 15  3. 6  4. \( m\angle A = 135; m\angle B = 45 \)  5. \( m\angle A = 10; m\angle B = 80 \)  6. \( m\angle PMQ = 125; m\angle QMN = 55 \)  7. \( m\angle BWC = m\angle CWD, m\angle AWB + m\angle BWC = 180; m\angle CWD + m\angle DWA = 180; m\angle AWB = m\angle AWD 

Guided Problem Solving 2-5
1. 90  2. See graph in Exercise 5 answer.  3. on the positive y-axis  4. Answers may vary. \( B \) can be any point on the positive y-axis. Sample: \( B(0, 3) \).

Guided Problem Solving 2-6
1. 90  2. See graph in Exercise 5 answer.  3. on the positive y-axis  4. Answers may vary. \( B \) can be any point on the positive y-axis. Sample: \( B(0, 3) \).

Guided Problem Solving 2-7
1. 90  2. See graph in Exercise 5 answer.  3. on the positive y-axis  4. Answers may vary. \( B \) can be any point on the positive y-axis. Sample: \( B(0, 3) \).
2A: Graphic Organizer
1. Reasoning and Proof  2. Answers may vary. Sample: conditional statements; writing biconditionals; converses; and using the Law of Detachment  3. Check students’ work.

2B: Reading Comprehension
1. 42 degrees  2. 38 degrees  3. b

2C: Reading/Writing Math Symbols
1. Segment MN is congruent to segment PQ.  2. If p, then q.  3. The length of MN is equal to the length of PQ.  4. Angle XQV is congruent to angle RDC.  5. If q, then p.  6. The measure of angle XQV is equal to the measure of angle RDC.  7. p if and only if q.  8. a → b  9. AB = MN  10. m∠XYZ = m∠RPS  11. b → a  12. AB ≡ MN  13. a ↔ b  14. ∠XYZ ≡ ∠RPS

2D: Visual Vocabulary Practice

2E: Vocabulary Check
Truth value: “True” or “false” according to whether the statement is true or false, respectively
Hypothesis: The part that follows if in an if-then statement.
Biconditional: The combination of a conditional statement and its converse; it contains the words “if and only if.”
Conclusion: The part of an if-then statement that follows then.
Conditional: An if-then statement.

2F: Vocabulary Review Puzzle

Chapter 3
Practice 3-1
1. corresponding angles  2. alternate interior angles  3. same-side interior angles  4. ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8  5. ∠4 and ∠6, ∠3 and ∠5  6. ∠4 and ∠5, ∠3 and ∠6  7. m∠1 = 100, alternate interior angles; m∠2 = 100, corresponding angles or vertical angles  8. m∠1 = 135, corresponding angles; m∠2 = 135, vertical angles  9. x = 103°, 77°, 103°  10. x = 30°, 85°, 85°

Guided Problem Solving 3-1
1. The top and bottom sides are parallel, and the left and right sides are parallel.  2. The two diagonals are transversals, and also each side of the parallelogram is a transversal for the two sides adjacent to it.  3. Corresponding angles, interior and exterior angles are formed.  4. v, w and x; By the Alternate Interior Angles Theorem, v = 42, w = 25 and x = 76.  5. Answers may vary. Possible answer: By the Same-Side Interior Angles Theorem, (w + 42) + (y + 76) = 180. Since w = 25, y = 37. (The two y’s are equal by Theorem 3-1.)  6. w = 25, y = 37, v = 42, x = 76; yes  7. v = 42, w = 35, x = 57, y = 46
Practice 3-2
1. \( \ell \) and \( m \), Converse of Same-Side Interior Angles Theorem
2. none
3. \( BC \) and \( AD \), Converse of Same-Side Interior Angles Theorem
4. \( BH \) and \( CT \), Converse of Corresponding Angles Postulate
5. 43 6. 90 7. 38 8. 100

Guided Problem Solving 3-2
1. \( \ell \) and \( m \)
2. transversals
3. \( x \)
4. the angles measuring
5. 19\( x \)+ and 17\( x \)
6. \( 180 - 19x = 17x \) or
7. \( 19x + 17x = 180 \)
8. \( x = 5 \)
9. With \( x = 5 \), \( 19x = 95 \) and
10. \( 17x = 85 \).

Practice 3-3
1. True. Every avenue will be parallel to Founders Avenue, and therefore every avenue will be perpendicular to Center City Boulevard, and therefore every avenue will be perpendicular to any street that is parallel to Center City Boulevard.
2. True. The fact that one intersection is perpendicular, plus the fact that every street belongs to one of two groups of parallel streets, is enough to guarantee that all intersections are perpendicular.
3. Not necessarily true. If there are more than three avenues and more than three boulevards, there will be some blocks bordered by neither Center City Boulevard nor Founders Avenue.
4. \( a \perp e \)
5. \( a \parallel e \)
6. \( a \parallel e \)
7. \( a \parallel e \)
8. If the number of \( \perp \) statements is even, then \( \ell_1 \parallel \ell_n \). If it is odd, then \( \ell_1 \perp \ell_n \).

Guided Problem Solving 3-3
1. supplementary angles
2. right angle
3. Any one of the following: Postulate 3-1, or Theorem 3-1, 3-2, 3-3 or 3-4
4. 90
5. It is congruent; Postulate 3-1
6. 90
7. \( a \perp e \)
8. It is true for any line parallel to \( b \).
9. Yes. The point is that a transversal cannot be perpendicular to just one of two parallel lines. It has to be perpendicular to both, or else to neither.

Practice 3-4
1. 125
2. 143
3. 129
4. 136
5. \( x = 35; y = 145; z = 25 \)
6. \( v = 118; w = 37; t = 62 \)
7. 50
8. 88
9. \( m \angle 1 = 33; m \angle 2 = 52 \)
10. right scalene
11. obtuse isosceles
12. equiangular equilateral

Guided Problem Solving 3-4
1. three
2. 180
3. right triangle
4. \( z = 90 \); Because it is given in the figure that \( BD \perp AC \).
5. Theorem 3-12, the Triangle Angle-Sum Theorem
6. \( x = 38 \)
7. \( y = 36 \)
8. \( \triangle ABD \) is a 36-54-90 right triangle. \( \triangle BCD \) is a 38-52-90 right triangle.
9. 74
10. \( \triangle ABC \) is a 52-54-74 acute triangle.
11. Yes, all three are acute angles, with \( \angle ABC \) visibly larger than \( \angle A \) and \( \angle C \).
12. \( \angle BCD \)

Practice 3-5
1. \( x = 120; y = 60 \)
2. \( n = 51\frac{1}{2} \)
3. \( a = 108; b = 72 \)
4. 109
5. 133
6. 129
7. 30
8. 150
9. 6
10. 5
11. \( BEDC \)
12. \( \angle FAE \)
13. \( \angle FAE \) and \( \angle BAE \)
14. \( ABCDE \)

Guided Problem Solving 3-5
1. A theater stage, consisting of a large platform surrounding a smaller platform. The shapes in the bottom part of the figure may represent a ramp for actors to enter and exit.
2. The measures of angles 1 and 2
3. Octagon
4. \((8 - 2)180 = 1080\) degrees
5. 135
6. 45
7. Yes, angle 1 is an obtuse angle and angle 2 is an acute angle.
8. trapezoids
9. 360

Practice 3-6
1. \( y = \frac{1}{3}x - 7 \)
2. \( y = -2x + 12 \)
3. \( y = \frac{4}{3}x - 2 \)
4. \( y = 4x - 13 \)

6.

7.

8. \( y = x + 4 \)
9. \( y = \frac{1}{2}x - 3 \)
10. \( y = -\frac{1}{2}x - \frac{1}{2} \)
11. \( y = -6x + 45 \)
12. \( y = -11; x = 2 \)
13. \( y = 2; x = 0 \)
14. By the way, \((0, -1)\) and \((-2, 0)\) are solutions of \( y = 4x - 1 \).
Guided Problem Solving 3-6

1. Graph showing triangle ABC with vertices A(0, 0), B(0, 6), and C(6, 0).

2. \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

3. \( y - y_1 = m(x - x_1) \)

4. Slope of \( \overrightarrow{AB} = \frac{6 - 0}{0 - 0} \) does not exist. The absolute values of the slopes are the same, but one slope is positive and the other is negative.

5. Point-slope form: \( y - 0 = \frac{5}{2}(x - 0) \)

6. Point-slope form: \( y = \frac{5}{2}x \)

7. Point-slope form: \( y - 5 = -\frac{5}{2}(x - 2) \) or \( y - 0 = -\frac{5}{2}(x - 4) \)

8. Of line \( \overrightarrow{BC} : (0, 10) \)

9. \( \triangle ABC \) appears to be an isosceles triangle, which is consistent with a horizontal base and two remaining sides having slopes of equal magnitude and opposite sign.

10. Slope of \( \overrightarrow{AB} : (0, 0) \)

Practice 3-7

1. neither; \( 3 \neq \frac{1}{3} \cdot \frac{1}{3} \neq -1 \)

2. perpendicular; \( \frac{1}{2} \cdot -2 = -1 \)

3. parallel; \( -\frac{2}{3} = -\frac{2}{3} \)

4. perpendicular; \( y = 2 \) is a horizontal line, \( x = 0 \) is a vertical line

5. perpendicular; \( -1 \cdot 1 = -1 \)

6. neither; \( \frac{1}{2} \neq -\frac{5}{2} \)

7. neither; \( \frac{9}{2} \neq 4, \frac{9}{2} \cdot 4 \neq -1 \)

8. parallel; \( \frac{1}{2} = \frac{1}{2} \)

9. \( y = \frac{2}{3}x \)

10. \( y = 2x - 4 \)

Guided Problem Solving 3-7

1. Graph showing a right triangle with vertices K(4, 0), L(0, 4), and M(0, 0).

2. A right angle

3. \( m_1 \cdot m_2 = -1 \)

4. Sides \( \overrightarrow{GH} \) and \( \overrightarrow{HK} \)

5. Slope of \( \overrightarrow{GH} = \frac{2}{5} \); slope of \( \overrightarrow{HK} = \frac{8}{3} \)

6. Product = \( -\frac{8}{5} \) \( \neq -1 \). Sides \( \overrightarrow{GH} \) and \( \overrightarrow{HK} \) are not perpendicular.

7. \( \triangle GHK \) has no pair of perpendicular sides. It is not a right triangle.

8. No \( \angle HKG \); approximately 80°

9. \( \angle GHK \)

10. Slope of \( \overrightarrow{LM} = \frac{7}{2} \) and slope of \( \overrightarrow{LN} = -\frac{2}{7} \). The product of the slopes is \(-1\), so \( \overrightarrow{LM} \) and \( \overrightarrow{LN} \) are perpendicular.

Practice 3-8

1. \( Q \)

2. \( T \)

3. Sample:

4. \( \overrightarrow{K} \)

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Guided Problem Solving 3-8
1. a line segment of length \( c \)  
2. Construct a quadrilateral with one pair of parallel sides of length \( c \), and then examine the other pair.  
3. The procedure is given on p. 181 of the text.  
4. Adjust the compass to exactly span line segment \( c \), end to end. Then tighten down the compass adjustment as necessary.  
5.  
6. They appear to be both congruent and parallel.  
8. The answers to Step 7 are confirmed.  
9. yes; a parallelogram

3A: Graphic Organizer
1. Parallel and Perpendicular Lines  
2. Answers may vary.  
Sample: properties of parallel lines; finding the measures of angles in triangles; classifying polygons; and graphing lines  
3. Check students’ work.

3B: Reading Comprehension
1. 14 spaces  
2. 6 spaces  
3. 60°  
4. corresponding  
5. $7000; $480  
6. the width of the stalls, 10 ft  
7. \( b \)

3C: Reading/Writing Math Symbols
1. \( m \perp n \)  
2. \( m\angle 1 + m\angle 2 = 180 \)  
3. \( AB \parallel CD \)  
4. \( m\angle MNQ + m\angle MNP = 90 \)  
5. \( \angle 3 \cong \angle EFD \)  
6. Line 1 is parallel to line 2.  
7. The measure of angle \( ABC \) is equal to the measure of angle \( XYZ \).  
8. Line \( AB \) is perpendicular to line \( DF \).  
9. Angle \( ABC \) and angle \( ABD \) are complementary.  
10. Angle 2 is a right angle, or the measure of angle 2 is 90°.  
11. Sample answer: \( \overrightarrow{CB}, m\angle BAF = m\angle GFA \)

3D: Visual Vocabulary Practice/High-Use Academic Words
1. property  
2. conclusion  
3. describe  
4. formula  
5. measure  
6. approximate  
7. compare  
8. contradiction  
9. pattern

3E: Vocabulary Check
Transversal: A line that intersects two coplanar lines in two points.  
Alternate interior angles: Nonadjacent interior angles that lie on opposite sides of the transversal.  
Same-side interior angles: Interior angles that lie on the same side of a transversal between two lines.  
Corresponding angles: Angles that lie on the same side of a transversal between two lines, in corresponding positions.  
Flow proof: A convincing argument that uses deductive reasoning, in which arrows show the logical connections between the statements.

3F: Vocabulary Review
1. C  
2. E  
3. D  
4. B  
5. A  
6. F  
7. K  
8. H  
9. L  
10. G  
11. I  
12. J
Chapter 4

Practice 4-1
1. \( m \angle 1 = 110; m \angle 2 = 120 \)
2. \( m \angle 3 = 90; m \angle 4 = 135 \)
3. \( \overline{CA} \parallel \overline{TS}, \overline{AT} \parallel \overline{SD}, \overline{CT} \parallel \overline{TD} \)
4. \( \angle C \equiv \angle I \), \( \angle A \equiv \angle S, \angle T \equiv \angle D \)
5. Yes; \( \triangle GHJ \equiv \triangle IHJ \) by Theorem 4-1 and the Reflexive Property of \( \equiv \). Therefore, \( \triangle GHJ \equiv \triangle IHJ \) by the definition of \( \equiv \) triangles.
6. No; \( \angle QSR \equiv \angle TSV \) because vertical angles are congruent, and \( \angle QTS \equiv \angle TSV \) by Theorem 4-1, but none of the sides are necessarily congruent. 7a. Given 7b. Vertical angles are \( \equiv \). 7c. Theorem 4-1 7d. Given 7e. Definition of \( \equiv \) triangles

Guided Problem Solving 4-1
1. right triangles 2. \( m \angle A = 45, m \angle B = m \angle L = 90 \), and \( AB = 4 \) in. 3. \( \triangle ABC \) is congruent to \( \triangle KLM \) means corresponding sides and angles are congruent. 4. \( x \) and \( t \)
5. 45 6. \( m \angle K = m \angle M = 45 \) 7. \( 3x = 45 \) 8. \( x = 15 \) 9. 4 10. \( 2t = 4 \) 11. \( t = 2 \) 12. The angle measures indicate that the two triangles are isosceles right triangles. This matches the appearance of the figure. 13. \( m \angle M = 60 \)

Practice 4-2
1. \( \triangle ADB \equiv \triangle CDB \) by SAS 2. not possible 3. \( \triangle TUS \equiv \triangle TXW \) by SSA, not possible 5. \( \triangle DEC \equiv \triangle GHF \) by SAS 6. \( \triangle PRN \equiv \triangle PRQ \) by SSS 7. \( \angle C \) 8. \( \overline{TA} \) and \( \overline{BC} \) 9. \( \angle A \) and \( \angle B \)
10. \( \triangle \equiv \triangle \) 11a. Given 11b. Reflexive Property of Congruence 11c. SAS Postulate

Guided Problem Solving 4-2
1. \( \overline{TSO} \) and \( \overline{SP} \) bisect \( \angle ISO \). 2. Prove whatever additional facts can be proven about \( \triangle ISP \) and \( \triangle OSP \), based on the given information. 3. \( \overline{TA} \equiv \overline{SO} \), \( \overline{IS} \equiv \overline{PS} \) 5. \( \overline{SP} \)
6. \( \triangle ISP \equiv \triangle OSP \) by Postulate 4-2, the Side-Angle-Side (SAS) Postulate 7. It does not matter. The Side-Angle-Side Postulate applies whether or not they are collinear. 8. It does follow; because \( \triangle ISP \equiv \triangle OSP \) and because \( \overline{TP} \) and \( \overline{PQ} \) are corresponding parts.

Practice 4-3
1. not possible 2. ASA Postulate 3. AAS Theorem 4. ASA Postulate 5. not possible 6. AAS Theorem 7. Statements
1. \( \triangle K \equiv \triangle M, \overline{KL} \equiv \overline{ML} \) 1. Given
2. \( \overline{JK} \equiv \overline{PL} \) 2. Vertical \( \angle s \) are \( \equiv \).
3. \( \triangle KJL \equiv \triangle PML \) 3. ASA Postulate
8. \( \overline{BC} \equiv \overline{EF} \), \( \angle KHJ \equiv \angle HKG \) or \( \angle KJH \equiv \angle HJG \)

Guided Problem Solving 4-3
1. Corresponding angles and alternate interior angles. 2. \( \angle EAB \) and \( \angle DBC \). 3. \( \angle EBA \) and \( \angle DCB \). 4. \( \angle EAB \) and \( \angle DBC \). 5. \( \angle EAB \equiv \angle DBC, \overline{AE} \equiv \overline{BD}, \) and \( \angle E \equiv \angle D \). 6. \( \angle EAB \equiv \angle DBC \) by Postulate 4-3, the Angle-Side-Angle (ASA) Postulate 7. Yes; Theorem 4-2, the Angle-Side-Angle (ASA) Theorem; \( \angle EBA \equiv \angle DCB \).
8. No, because now there is no way to demonstrate a second pair of congruent sides, nor a second pair of congruent angles.

Practice 4-4
1. \( \overline{BD} \) is a common side, so \( \triangle ADB \equiv \triangle CDB \) by SAS, and \( \angle A \equiv \angle C \) by CPCTC. 2. \( DF \) is a common side, so \( \triangle QHE \equiv \triangle HFG \) by SAS, and \( \angle HE \equiv \angle FG \) by CPCTC.
3. \( QS \) is a common side, so \( \triangle QTS \equiv \triangle SRQ \) by AAS. 4. \( \angle QST \equiv \angle SQR \) by CPCTC. 4. \( \angle ZAY \) and \( \angle CAB \) are vertical angles, so \( \triangle ABC \equiv \triangle AYZ \) by ASA, \( \overline{ZA} \equiv \overline{AC} \) by CPCTC.
5. \( \angle JKH \) and \( \angle KLM \) are vertical angles, so \( \angle HJK \equiv \angle MLK \) by AAS, and \( \angle I \equiv \angle K \) by CPCTC. 6. \( PR \) is a common side, so \( \triangle PNR \equiv \triangle QRP \) by SSS, and \( \angle AP \equiv \angle Q \) by CPCTC.
7. First, show that \( \angle ACB \) and \( \angle ECD \) are vertical angles. Then, show \( \triangle ABC \equiv \triangle EDC \) by ASA. Last, show \( \angle A \equiv \angle E \) by CPCTC.

Guided Problem Solving 4-4
1. A compass with a fixed setting was used to draw two circular arcs, both centered at point \( P \) but crossing \( \ell \) in different locations, which were labeled \( A \) and \( B \). The compass was used again, with a wider setting, to draw two intersecting circular arcs, one centered at \( A \) and one at \( B \). The point at which the new arc intersected was labeled \( C \). Finally, line \( CP \) was drawn. 2. Find equal lengths or distances and explain why \( CP \) is perpendicular to \( \ell \).
3. \( \angle ACP \) and \( \angle BCP \)
4. \( AP = PB \) and \( AC = BC \). 5. \( \triangle APC \equiv \triangle BPC \), by Postulate 4-1, the Side-Side-Side (SSS) Postulate 6. \( \triangle APC \equiv \triangle BPC \) by CPCTC 7. Since \( \angle APC \equiv \angle BPC \), \( m \angle APC = m \angle BPC \), and \( m \angle APC + m \angle BPC = 180 \), it follows that \( m \angle APC = m \angle BPC = 90 \).
8. From the definition of perpendicular and the fact that \( m \angle APC = m \angle BPC = 90 \) the distances do not matter, so long as \( AP = BP \) and \( AC = BC \). That is what is required in order that \( \triangle APC \equiv \triangle BPC \). 9. Draw a line, and use the construction technique of the problem to construct a second line perpendicular to the first. Then do the same thing again to construct a third line perpendicular to the second line. The first and third lines will be parallel, by Theorem 3-10.

Practice 4-5
1. \( x = 35; y = 35 \) 2. \( x = 80; y = 90 \) 3. \( t = 150 \)
4. \( x = 55; y = 70; z = 125 \) 5. \( x = 6 \) 6. \( z = 120 \)
7. \( \overline{AD} \equiv \overline{DE} \equiv \overline{EF} \) 8. \( \overline{KT} \equiv \angle KLT \equiv \angle KJI \) 9. \( \overline{BA} \equiv \angle ABI \) 10. \( \overline{ABJ} \equiv \angle AJB \) 11. \( 130 \) 12. \( x = 70; y = 55 \)

Guided Problem Solving 4-5
1. One angle is obtuse. The other two angles are acute and congruent. 2. Highlight an obtuse isosceles angle and find its angle measures, then find all the other angle measures represented in the figure.
Practice 4-6

Statements
1. \( AB \perp BC, \quad DE \perp FE \)
2. \( \angle B, \angle E \) are right \( \angle s. \)
3. \( AC \cong FD, \quad AB \cong ED \)
4. \( \triangle ABC \cong \triangle DEF \)

Reasons
1. Given
2. Perpendicular lines form right \( \angle s. \)
3. Given
4. HL Theorem

Guided Problem Solving 4-6
1. Two congruent right triangles. Each one has a leg and a hypotenuse labeled with a variable expression. 2. The values of \( x \) and \( y \) for which the triangles are congruent by HL. 3. The two shorter legs are congruent. 4. \( x = y + 1 \) 5. The hypotenuses are congruent. 6. \( x + 3 = 3y \) 7. \( x = 5; \quad y = 2 \)

Guided Problem Solving 4-7
1. The figure, a list of parallel and perpendicular pairs of sides, and one known angle measure, namely \( m \angle A = 56 \), 2. nine 3. They are congruent and have equal measures. 4. \( m \angle A = m \angle 1 = m \angle 2 = 56 \) 5. \( m \angle 4 = 90 \)
6. \( m \angle 3 = 34 \) 7. \( m \angle DCE = 58 \) 8. \( m \angle 5 = 22 \)
9. \( m \angle FGC = 90 \) 10. \( m \angle 6 = 34 \) 11. \( m \angle 7 = 34 \), \( m \angle 8 = 68, \) and \( m \angle 9 = 112 \) 12. \( m \angle 9 = 56 + 56 = 112 \)
13. \( m \angle FIC = 180 - (m \angle 2 + m \angle 3) = 90 \); \( m \angle DHC = m \angle A = 90; m \angle FIC = 180 - m \angle 9 = 68; m \angle BIG = m \angle FIC = 90 \)

4A: Graphic Organizer
1. Congruent Triangles 2. Answers may vary. Sample: congruent figures; triangle congruence by SSS, SAS, ASA, and AAS; proving parts of triangles congruent; the Isosceles Triangle Theorem 3. Check students’ work.

4B: Reading Comprehension
1. Yes. Using the Isosceles Triangle Theorem, \( \angle W \equiv \angle Y \). It is given that \( WX \cong XY \) and \( WU \cong YV \). Therefore \( \triangle WUX \cong \triangle YVX \) by SAS. 2. There is not enough information. You need to know if \( AC \cong EC \), if \( \angle A \equiv \angle E \), or if \( \angle B \equiv \angle D \). 3. a
4C: Reading/Writing Math Symbols
1. Angle-Angle-Side  2. triangle XYZ  3. angle PQR

4D: Visual Vocabulary Practice

4E: Vocabulary Check
Angle: Formed by two rays with the same endpoint.
Congruent angles: Angles that have the same measure.
Congruent segments: Segments that have the same length.
Corresponding polygons: Polygons that have corresponding sides congruent and corresponding angles congruent.
CPCTC: An abbreviation for “corresponding parts of congruent triangles are congruent.”

4F: Vocabulary Review Puzzle
10. parallel  11. corresponding

Chapter 5

Practice 5-1
1a. 8 cm  1b. 16 cm  1c. 14 cm  2a. 9.5 cm  2b. 17.5 cm
2c. 14.5 cm  3. 17  4. 7  5. 42  6. 16.5  7a. 18  7b. 61
8. $PR \parallel YZ$, $PQ \parallel XZ$, $XY \parallel RO$

Guided Problem Solving 5-1
1. 30 units  2. The sides of the large triangle are each bisected by intersections with the two line segments lying in the interior of the large triangle. 3. The value of x  4. They are called midsegments. 5. They are parallel, and the side labeled 30 is half the length of the side labeled x. 6. $x = 60$
7. Yes; the side labeled x appears to be about twice as long as the side labeled 30. 8. No. Those lengths are not fixed by the given information. (The triangle could be vertically stretched or shrunk without changing the lengths of the labeled sides.) All one can say is that the midsegment is half as long as the side it is parallel to.

Practice 5-2
1. $WY$ is the perpendicular bisector of $XZ$. 2. 4  3. 9
4. right triangle  5. 5  6. 17  7. isosceles triangle  8. 3.5
9. 21  10. right triangle  11. $JP$ is the bisector of $\angle L$IN
12. 9  13. 45  14. 14  15. right isosceles triangle

Guided Problem Solving 5-2
1. See answer to Step 1, above. 2. See answer to Step 1, above. 3. Plot a point and explain why it lies on the bisector of the angle at the origin. 5. line $l$: $y = -\frac{3}{4}x + \frac{25}{2}$; line $m$: $x = 10$
6. $C(10.5)$ 7. $CA = CB = 5$; yes 8. Theorem 5-5, the Converse of the Angle Bisector Theorem 9. $m\angle AOC = m\angle BOC = 27$ 10. Draw $l$, $m$, and $C$, then draw $OC$. Since $OA = OB = 10$, it follows that $\triangle OAC \cong \triangle OBC$, by $HL$. Then $CA \cong CB$ and $\angle AOC \cong \angle BOC$ by CPCTC.

Practice 5-3
1. $(-2, 2)$ 2. $(4, 0)$ 3. altitude  4. median
5. perpendicular bisector  6. angle bisector
7a. $(2, 0)$ 7b. $(-2, -2)$ 8a. $(0, 0)$ 8b. $(3, -4)$

Guided Problem Solving 5-3
1. the figure and a proof with some parts left blank  2. Fill in the blanks. 3. $AB$ 4. Theorem 5-2, the Perpendicular Bisector Theorem 5. $BC$; $XC$ 6. The Transitive Property of Equality 7. Perpendicular Bisector. (This converse is Theorem 5-3.) 8. The point of the proof is to demonstrate that $n$ runs through point $X$. It would not be appropriate to show that fact as already given in the figure. 9. Nothing essential would change. Point $X$ would lie outside $\triangle ABC$ (below $BC$), but the proof would run just the same.

Practice 5-4
1. I and III  2. I and II  3. The angle measure is not 65.
4. Tina does not have her driver’s license. 5. The figure does not have eight sides. 6. $\triangle ABC$ is congruent to $\triangle XYZ$.
7a. If you do not live in Toronto, then you do not live in Canada; false. 7b. If you do not live in Canada, then you do not live in Toronto; true. 8. Assume that $m\angle A \neq m\angle B$. 9. Assume that $LM$ does not intersect $NO$. 10. Assume that it is not sunny outside. 11. Assume that $m\angle A \geq 90$. This means that $m\angle A + m\angle C \geq 180$. This, in turn, means that the sum of the angles of $\triangle ABC$ exceeds 180, which contradicts the Triangle Angle-Sum Theorem. So the assumption that $m\angle A \geq 90$ must be incorrect. Therefore, $m\angle A < 90$.

Guided Problem Solving 5-4
1. Ice is forming on the sidewalk in front of Toni’s house.
2. Use indirect reasoning to show that the temperature of the sidewalk surface must be 32°F or lower. 3. The temperature
of the sidewalk in front of Toni’s house is greater than 32°F.  
4. Water is liquid (ice does not form) above 32°F.  
5. There is no ice forming on the sidewalk in front of Toni’s house.  
6. The result from step 5 contradicts the information identified as given in step 1.  
7. The temperature of the sidewalk in front of Toni’s house is less than or equal to 32°F.  
8. If the temperature is above 32°F, water remains liquid. This is reliably true. Converse: If water remains liquid, the temperature is above 32°F. This is not reliably true. Adding salt will cause water to remain liquid even below 32°F.

9. Suppose two people are each the world’s tallest person. Call them person A and person B. Then person A would be taller than everyone else, including B, but by the same token B would be taller than A. It is a contradiction for two people each to be taller than the other. So it is impossible for two people each to be the World’s Tallest Person.

Practice 5-5
1. \( \angle M, \angle N \)  
2. \( \angle C, \angle D \)  
3. \( \angle R, \angle P \)  
4. \( \angle A, \angle T \)  
5. yes; \( 4 + 7 > 8, 7 + 8 > 4, 8 + 4 > 7 \)  
6. no; \( 6 + 10 > 17 \)  
7. yes; \( 4 + 4 > 4 \)  
8. yes; \( 11 + 12 > 13, 12 + 13 > 11, 13 + 11 > 12 \)  
9. no; \( 18 + 20 \not> 40 \)  
10. no; \( 1.2 + 2.6 \not> 4.9 \)  
11. \( BO, BT, LO \)  
12. \( RS, ST, RT \)  
13. \( \angle D, \angle S, \angle A \)  
14. \( \angle N, \angle S, \angle J \)  
15. \( 3 < x < 11 \)  
16. \( 8 < x < 26 \)  
17. \( 0 < x < 10 \)  
18. \( 9 < x < 31 \)

Guided Problem Solving 5-5
1. 

2. The side opposite the larger included angle is greater than the side opposite the smaller included angle.  
3. The angle opposite the larger side is greater than the angle opposite the smaller side.  
4. The opposite sides each have a length of nearly the sum of the other two side lengths.  
5. The opposite sides are the same length. They are corresponding parts of triangles that are congruent by SAS.

5A: Graphic Organizer
1. Relationships Within Triangles  
2. Answers may vary: Sample: midsegments of triangles; bisectors in triangles; concurrent lines, medians, and altitudes; and inverses, contrapositives, and indirect reasoning  
3. Check students’ work.

5B: Reading Comprehension
1. The width of the tar pit is 10 meters.  
2. b

5C: Reading/Writing Math Symbols
1. L  
2. F  
3. O  
4. G  
5. A  
6. I  
7. M  
8. H  
9. E  
10. K  
11. B  
12. D  
13. N  
14. J  
15. C

5D: Visual Vocabulary Practice
1. median  
2. negation  
3. circumcenter  
4. contrapositive  
5. centroid  
6. equivalent statements  
7. incenter  
8. inverse  
9. altitude

5E: Vocabulary Check
Midpoint: A point that divides a line segment into two congruent segments.  
Midsegment of a triangle: The segment that joins the midpoints of two sides of a triangle.  
Proof: A convincing argument that uses deductive reasoning.  
Coordinate proof: A proof in which a figure is drawn on a coordinate plane and the formulas for slope, midpoint, and distance are used to prove properties of the figure.  
Distance from a point to a line: The length of the perpendicular segment from the point to the line.

5F: Vocabulary Review
1. altitude  
2. line  
3. median  
4. negation  
5. contrapositive  
6. incenter  
7. orthocenter  
8. slope-intercept  
9. exterior  
10. obtuse  
11. alternate interior  
12. centroid  
13. equivalent  
14. right  
15. parallel

Chapter 6

Practice 6-1
1. parallelogram  
2. rectangle  
3. quadrilateral  
4. kite, quadrilateral  
5. trapezoid, isosceles trapezoid, quadrilateral  
6. square, rectangle, parallelogram, rhombus, quadrilateral  
7. \( x = 7; AB = BD = DC = CA = 11 \)  
8. \( f = 5; g = 11; FG = GH = HI = IF = 17 \)  
9. parallelogram  
10. kite

Guided Problem Solving 6-1
1. a labeled figure, which shows an isosceles trapezoid  
2. The nonparallel sides are congruent.  
3. The measures of the angles and the lengths of the sides  
4. \( m \angle G = c \)  
5. \( c + (4c - 20) = 180 \)  
6. 40  
7. \( m \angle D = m \angle G = 40 \)  
8. \( a - 4 = 11 \)  
9. 15  
10. \( DE = FG = 11, EF = 15, DG = 32 \)  
11. \( 40 + 40 + 140 + 140 = 360 = (4 - 2)180 \)  
12. \( m \angle D = m \angle G = 39, m \angle E = m \angle F = 141 \)

Practice 6-2
1. 15  
2. 32  
3. 7  
4. 12  
5. 9  
6. 8  
7. 100  
8. 40; 140; 40  
9. 113; 45; 22  
10. 115; 15; 50  
11. 55; 105; 55  
12. 32; 98; 50  
13. 16  
14. 35  
15. 28
Guided Problem Solving 6-2
1. the ratio of two different angle measures in a parallelogram.
2. The consecutive angles are supplementary.
3. \[ \begin{align*}
9x
\end{align*} \]
4. the measures of the angles. 5. The angles are supplementary angles, because they are consecutive.
6. \( x + 9x = 180 \) 7. 18 and 162. 8. No; the lengths of the sides are irrelevant in this problem. 9. 30 and 150

Practice 6-3
1. no 2. yes 3. yes 4. yes 5. \( x = 2; y = 3 \) 6. \( x = 64; y = 10 \) 7. \( x = 8; \) the figure is a \( \square \) because both pairs of opposite sides are congruent. 8. \( x = 25; \) the figure is a \( \square \) because the congruent opposite sides are \( \parallel \) by the Converse of the Alternate Interior Angles Theorem. 9. No; the congruent opposite sides do not have to be \( \parallel \). 10. No; the figure could be a trapezoid. 11. Yes; both pairs of opposite sides are congruent. 12. Yes; both pairs of opposite sides are \( \parallel \) by the converse of the Alternate Interior Angles Theorem. 13. No; only one pair of opposite angles is congruent. 14. Yes; one pair of opposite sides is both congruent and \( \parallel \).

Guided Problem Solving 6-3
1. a labeled figure, which shows a quadrilateral that appears to be a parallelogram. 2. The consecutive angles are supplementary. 3. find values for \( x \) and \( y \) which make the quadrilateral a parallelogram. 4. \( m \angle A + m \angle D = 180 \), so that \( \angle A \) and \( \angle D \) meet the requirements for same-side interior angles on the transversal of two parallel lines (Theorems 3-2 and 3-6). 5. \( \angle B \equiv \angle D \), by Theorem 6-2. 6. \( x + 10 + 5y = 180; 8x + 5 = 5y \) 7. \( x = 15, y = 25 \) 8. \( m \angle A = m \angle C = 55 \) and \( m \angle B = m \angle D = 125 \), which matches the appearance of the figure. 9. \( (3x + 10) + (8x + 5) = 180 \); yes

Practice 6-4
1a. rhombus 1b. 72; 54; 47 2a. rectangle 2b. 37; 53; 106; 74 3a. rectangle 3b. 60; 30; 60; 30 4a. rhombus 4b. 22; 68; 68; 90 5. Possible; opposite angles are congruent in a parallelogram. 6. Impossible; if the diagonals are perpendicular, then the parallelogram should be a rhombus, but the sides are not of equal length. 7. \( x = 7; HJ = 7; IK = 7 \) 8. \( x = 6; HJ = 25; IK = 25 \) 9a. 90; 90; 29; 29 9b. 288 cm² 10a. 38; 90; 38 10b. 260 m²

Guided Problem Solving 6-4
1. A labeled figure, which shows a parallelogram. One angle is a right angle, and two adjacent sides are congruent. Algebraic expressions are given for the lengths of three line segments. 2. diagonals 3. Find the values of \( x \) and \( y \). 4. It is a triangle. Theorem 6-1 and the fact that \( \overline{AB} \equiv \overline{AD} \) imply that all four sides are congruent. Theorems 3-11 and 6-2 plus the fact that \( m \angle B = 90 \) imply that all four angles are right angles.

Guided Problem Solving 6-5
1. Isosceles trapezoid \( ABCD \) with \( \overline{AB} \equiv \overline{CD} \) and \( \angle BAD = \angle D \). \( \overline{AB} \equiv \overline{CD} \) is Given. \( \overline{DC} \equiv \overline{AE} \) because opposite sides of a parallelogram are congruent (Theorem 6-1). \( \overline{AB} \equiv \overline{AE} \) is from the Transitive Property of Congruency. 4. Isosceles; \( \equiv \); because base angles of an isosceles triangle are congruent. 5. \( \angle 1 \equiv \angle 3 \) because corresponding angles on a transversal of two parallel lines are congruent. 6. \( \angle B \equiv \angle C \) by the Transitive Property of Congruency. 7. \( \angle BAD \) is a same-side interior angle with \( \angle B \), and \( \angle D \) is a same-side interior angle with \( \angle C \). 8. This is not a problem, because for \( AD > BC \) there is a similar proof with a line segment drawn from \( B \) to a point \( E \) lying on \( \overline{AD} \). 9. The two drawn segments can be shown to be congruent, and then one has two congruent right triangles by the HL Theorem. \( \angle B \equiv \angle C \) follows by CPCTC and \( \angle BAD \equiv \angle D \) because they are supplements of congruent angles.

Practice 6-6
1. \( (1.5a, 2b); a \) 2. \( (0.5a, 0); a \) 3. \( (0.5a, b); \sqrt{a^2 + 4b^2} \) 4. \( 0 \) 5. \( 1 \) 6. \( -\frac{1}{2} \) 7. \( \frac{2b}{5a} \) 8. \( \frac{2b}{5a} \) 9. \( E(a, 3b); I(4a, 0) \) 10. \( D(4a, b); I(3a, 0) \) 11. \( (3a, 0) \) 12. \( (b, 0) \)

Guided Problem Solving 6-6
1. a rhombus with coordinates given for two vertices. 2. A rhombus is a parallelogram with four congruent sides. 3. the coordinates of the other two vertices. 4. They are the diagonals. 5. They bisect each other. 6. \( W(-2r, 0) \), \( Z(0, -2t) \) 7. No; neither Theorem 6-3 nor any other theorem or result would apply. 8. Slope of \( \overline{WX} = 0 \) and slope of \( \overline{YZ} \) is undefined. This confirms Theorem 6-10, which says that the diagonals of a rhombus are perpendicular.

Practice 6-7
1a. \( \frac{p}{q} \) 1b. \( y = mx + b; q = \frac{p}{q}(y) + b; b = q - \frac{p^2}{q}; \) \( y = \frac{p}{q}x + q - \frac{p^2}{q} \) 1c. \( x = r + p \) 1d. \( y = \frac{p}{q}(r + p) + q - \frac{p^2}{q}; y = \frac{p}{q}(r + p) + q; \) intersection at \( (r + p, \frac{p}{q} + q) \) 1e. \( \frac{q}{p} \) 1f. \( \frac{p}{q} \) 1g. \( y = mx + b; q = \frac{p}{q}(r + b); b = q - \frac{p^2}{q}; y = \frac{p}{q}x + q - \frac{p^2}{q}; \) 1h. \( y = \frac{p}{q}(r + p) + q - \frac{p^2}{q}; y = \frac{p}{q}x + \frac{p}{q} + \frac{p^2}{q} + q - \frac{p^2}{q}; \)
Guided Problem Solving 6-7

1. kite \( DEFG \) with \( DE = EF \) with the midpoint of each side identified
2. A kite is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent.
3. The midpoints are the vertices of a rectangle.
4. \( D(-2b, 2c), G(0, 0) \) 5. \( L(b, a + c), M(b, c), N(-b, c) \) 6. Slope of \( KL \) = slope of \( NM \) = 0, slopes of \( KN \) and \( LM \) are undefined
7. Opposite sides are parallel; it is a rectangle.
8. Adjacent sides are perpendicular.
9. right angles
10. Answers will vary. Example: \( a = 3, b = 2, c = 2 \) yields the points \( D(-4, 4), E(0, 6), F(4, 4), G(0, 0) \) with midpoints at \( (-2, 2), (-2, 5), (2, 5), \) and \( (2, 2) \).
Connecting these midpoints forms a rectangle.
11. Construct \( DF \) and \( EG \). Slope of \( DF \) = 0, so \( DF \) is horizontal. Slope of \( EG \) is undefined, so \( EG \) is vertical.

6A: Graphic Organizer
1. Quadrilaterals
2. Answers may vary. Sample: classifying quadrilaterals; properties of parallelograms; proving that a quadrilateral is a parallelogram; and special parallelograms
3. Check students’ work.

6B: Reading Comprehension
1. \( QT \equiv SR \) \( QR \equiv ST \) \( QT \parallel RS \) \( QR \parallel TV \)
2. No, it cannot be proven that \( \triangle QTV \equiv \triangle SRU \) because with the given information, only one side and one angle of the two triangles can be proven to be congruent. Another side or angle is needed. If it were given that \( QUSV \) is a parallelogram, then the proof could be made.
3. All four sides are congruent.
4. Yes. Since \( EG \equiv EG \) by the Reflexive Property, \( \triangle EFG \equiv \triangle EHG \) by SSS.
5. b

6C: Reading/Writing Math Symbols
1. \( \times, \div, +, - \)
2. No, it cannot be proven that \( n_{QTV} > n_{SRU} \) because with the given information, only one side and one angle of the two triangles can be proven to be congruent. Another side or angle is needed. If it were given that \( QUSV \) is a parallelogram, then the proof could be made.
3. All four sides are congruent.
4. Yes. Since \( \triangle EFG \equiv \triangle EHG \) by SSS.

6D: Visual Vocabulary Practice/High-Use Academic Words
1. solve
2. deduce
3. equivalent
4. indirect
5. equal
6. analysis
7. identify
8. convert
9. common

6E: Visual Vocabulary Check
Consecutive angles: Angles of a polygon that share a common side.
Kite: A quadrilateral with two pairs of congruent adjacent sides and no opposite sides congruent.
Parallelogram: A quadrilateral with two pairs of parallel sides.
Rhombus: A parallelogram with four congruent sides.
Trapezoid: A quadrilateral with exactly one pair of parallel sides.

6F: Vocabulary Review Puzzle
Guided Problem Solving 7-1
1. ratios 2. \( \frac{42}{29,000} = \frac{1}{1,000,000} \) 3. the denominator 4. \( \frac{x}{29,000} = \frac{1}{1,000,000} \)

Guided Problem Solving 7-2
1. \( \triangle ABC \sim \triangle XYZ \), with similarity ratio 2 : 1
2. Not similar; corresponding sides are not proportional.
3. Not similar; corresponding angles are not congruent.
4. \( \triangle ABC \sim \triangle KMN \), with similarity ratio 4 : 7
5. \( \angle I \) 6. \( \angle O \) 7. \( \angle NO \) 8. \( \angle LO \) 9. 3.96 ft 10. 3.75 cm
11. \( \frac{2}{3} \) 12. 53 13. 7 14. 16. 15. 16. 37

Guided Problem Solving 7-3
1. \( \angle AXB \equiv \angle RXQ \) because vertical angles are \( \equiv \). \( \angle A \equiv \angle R \) (Given). Therefore \( \triangle AXB \sim \triangle RXQ \) by the AA ~ Postulate.
2. Because \( \frac{MP}{PQ} = \frac{QX}{MR} = \frac{XW}{AL} = \frac{3}{4} \), \( \triangle MPX \sim \triangle LWA \) by the SSS \ (~ Similarity) Theorem.
3. \( \angle QMP \equiv \angle AMB \) because \( \angle s \) are \( \equiv \). Then, because \( \frac{QM}{AM} = \frac{PM}{BM} = \frac{2}{3} \), \( \triangle QMP \sim \triangle AMB \) by the SAS \ (~ Similarity) Theorem.
4. Because \( AX = BX \) and \( CX = RX \), \( \frac{AX}{MS} = \frac{AX}{RX} \) \( \angle AXB \equiv \angle CXR \) because vertical angles are \( \equiv \). Therefore \( \triangle AXB \sim \triangle CXR \) by the SAS \ (~ Similarity) Theorem.
5. \( \frac{15}{2} \) 6. 2 7. \( \frac{20}{3} \) 8. 20 9. 33 ft

Guided Problem Solving 7-4
1. \( \triangle ABC, \triangle ACD, \triangle BCD \)
2. 1 3. 1 4. 1 5. 1
6. 2; 1 7. 2 8. \( \sqrt{2} \) 9. \( \sqrt{2} \) 10. yes 11. no (This would require the Pythagorean Theorem.)

Practice 7-5
1. \( BE \) 2. \( BC \) 3. \( JD \) 4. \( BE \) 5. \( \frac{16}{3} \) 6. 4 7. \( x = \frac{25}{37} \)
8. \( \frac{15}{4} \) 9. \( x = 6 \); 6 10. 2 11. 10

Guided Problem Solving 7-5
1. parallel 2. \( CE; BD \) 3. 6:15 4. 90 5. yes 6. yes 7. The sides would not be parallel. 8. They are similar triangles.

7A: Graphic Organizer
1. Similarity 2. Answers may vary. Sample: ratios and proportions; similar polygons; proving triangles similar; and similarity in right triangles
3. Check students’ work.

7B: Reading Comprehension
1. \( \triangle DHB \sim \triangle ACD \) 2. AA similarity postulate
The triangles have two similar angles. 3. 1 : 2 4. 1 : 2
5. \( \frac{DB}{AB} = \frac{HB}{CB} \) 6. 250 ft 7. a

7C: Reading/Writing Math Symbols
1. no 2. no 3. yes 4. no 5. yes 6. no 7. yes, AAS
8. yes, Hypotenuse-Leg Theorem 9. yes, SAS or ASA or AAS 10. not possible

7D: Visual Vocabulary Practice
1. Angle-Angle Similarity Postulate 2. golden ratio
3. Side-Side-Side Similarity Theorem 4. geometric mean
5. scale 6. Cross-Product Property 7. golden rectangle
8. simplest radical form 9. Side-Angle-Side Similarity Theorem

7E: Vocabulary Check
Similarity ratio: The ratio of lengths of corresponding sides of similar polygons.
Cross-Product Property: The product of the extremes of a proportion is equal to the product of the means.
Ratio: A comparison of two quantities by division.
Golden rectangle: A rectangle that can be divided into a square and a rectangle that is similar to the original rectangle.
Scale: The ratio of any length in a scale drawing to the corresponding actual length.

7F: Vocabulary Review
Chapter 8

Practice 8-1
1. \( \sqrt{51} \) 2. \( 2\sqrt{65} \) 3. \( 2\sqrt{21} \) 4. \( 18\sqrt{2} \) 5. 46 in. 6. 78 ft 7. 279 cm 8. 19 m 9. acute 10. obtuse 11. right

Guided Problem Solving 8-1
1. the sum of the lengths of the sides 2. Pythagorean Theorem 3. 7 cm 4. 4 cm \( \times \) 3 cm 5. \( c^2 = 4^2 + 3^2 \) 6. 5 7. 12 cm 8. perimeter of rectangle = 14 cm; yes 9. Answers will vary; example: Draw a 4 cm \( \times \) 3 cm grid, copy the given figure, measure the lengths with a ruler, add them together. 10. 20 cm

Practice 8-2
1. \( x = 2; y = \sqrt{3} \) 2. \( 8\sqrt{2} \) 3. \( 14\sqrt{2} \) 4. 2 5. \( x = 15; y = 15\sqrt{3} \) 6. \( 3\sqrt{2} \) 7. 42 cm 8. 10.4 ft, 12 ft 9. \( a = 4; b = 3 \) 10. \( p = 4\sqrt{3}; q = 4\sqrt{3}; r = 8; s = 4\sqrt{6} \)

Guided Problem Solving 8-2
1. 30\(^\circ\)-60\(^\circ\)-90\(^\circ\) triangle 2. \( l \) 3. \( h \) 4. \( \sqrt{3} \) 5. \( \frac{3\sqrt{3}}{2} \) or 8\( \sqrt{3} \) 6. 2 7. \( \frac{48}{\sqrt{3}} \) or 16\( \sqrt{3} \) 8. 28 ft 9. 0.28 min 10. yes 11. 34 ft

Practice 8-3
1. \( \tan E = \frac{2}{3}; \tan F = \frac{5}{2} \) 2. tan \( E = \frac{2}{3}; \tan F = \frac{5}{2} \) 3. 12.4 4. 31\(^\circ\) 5. 7.1 6. 6.4 7. 26.6 8. 71.6 9. 39 10. 72 11. 39 12. 54

Guided Problem Solving 8-3
1. 

\[ \begin{align*} A & \quad 20 \text{ cm} \\ 80 \text{ cm} \end{align*} \]

2. 180 3. \( m\angle A = 2m\angle X \) 4. 90 5. base: 40 cm, height: 10 cm 6. 4 7. 4 8. 76 9. 152 10. 28 11. yes 12. 46

Practice 8-4
1. \( \sin P = \frac{2\sqrt{10}}{5}; \cos P = \frac{3}{5} \) 2. \( \sin P = \frac{4}{5}; \cos P = \frac{3}{5} \) 3. \( \sin P = \frac{\sqrt{11}}{6}; \cos P = \frac{5}{6} \) 4. \( \sin P = \frac{15}{17}; \cos P = \frac{8}{17} \) 5. 64 6. 11.0 7. 7.0 8. 7.8 9. 53 10. 6.6 11. 11.0 12. 11.5

Guided Problem Solving 8-4
1. The sides are parallel. 2. sine 3. \( \sin 30\(^\circ\) = \frac{w}{6} \) 4. 3.0 5. yes 6. cosine 7. \( \cos x^\circ = \frac{3}{4} \) 8. 41 9. Answers may vary. Sample: \( \cos 60\(^\circ\) \geq \frac{3}{6}; \sin 49\(^\circ\) \geq \frac{3}{4} \) 10. 5.2, 2.6

Practice 8-5
1a. angle of depression from the plane to the person 1b. angle of elevation from the person to the plane 1c. angle of depression from the person to the sailboat 1d. angle of elevation from the sailboat to the person 2. 116.6 ft 3. 84.8 ft 4. 46.7 ft 5. 31.2 yd 6a.

6b. 26 ft

Guided Problem Solving 8-5
1. \( \angle e = 1; \angle d = \angle c \) 2. congruent 3. \( m\angle e = m\angle d \) 4. \( 7x - 5 = 4(x + 7) \) 5. 11 6. 72 7. 72 8. yes 9. 44, 44

Practice 8-6
1. (46.0, 46.0) 2. (89.2, -80.3) 3. 38.6 mi/h; 31.2\(^\circ\) north of east 4. 134.5 m; 42.0\(^\circ\) south of west 5. 55\(^\circ\) north of east 6. 33\(^\circ\) west of north 7a. (1, 5) 7b.

8a. (1, -1) 8b.

9. Sample:

\[ \begin{align*} W & \quad 48^\circ \\ N \end{align*} \]
Guided Problem Solving 8-6
1. Check students’ work.
2. \( \frac{x}{100} = \frac{y}{100} \)
3. 100 cos 30°; 100 sin 30°
4. 86.6; 50
5. 86.6, 50
6. 86.6, -50
7. (173.2, 0)
8. 173; due east
9. yes
10. 100; due east

8A: Graphic Organizer
1. Right Triangles and Trigonometry
2. Answers may vary.
   Sample: the Pythagorean Theorem; special right triangles; the tangent ratio; sine and cosine ratios; angles of elevation and depression; vectors
3. Check students’ work.

8B: Reading Comprehension
1. A
2. J
3. B
4. J
5. B
6. B
7. b

8C: Reading/Writing Math Symbols
1. F
2. G
3. D
4. A
5. C
6. B
7. H
8. E
9. \( \sin^{-1} A = \frac{5}{12} \)
10. \( \triangle ABC \sim \triangle XYZ \)
11. \( m \angle A \approx 52° \)
12. \( \tan Z = \frac{7}{24} \)

8D: Visual Vocabulary Practice
1. 30°-60°-90° triangle
2. inverse of tangent
3. congruent sides
4. tangent
5. Pythagorean Theorem
6. hypotenuse
7. 45°-45°-90° triangle
8. Pythagorean triple
9. obtuse triangle

8E: Vocabulary Check
Obtuse triangle: A triangle with one angle whose measure is between 90 and 180.
Isosceles triangle: A triangle that has at least two congruent sides.
Hypotenuse: The side opposite the right angle in a right triangle.
Right triangle: A triangle that contains one right angle.
Pythagorean triple: A set of three nonzero whole numbers \( a, b, \) and \( c \) that satisfy the equation \( a^2 + b^2 = c^2 \).

8F: Vocabulary Review Puzzle
Chapter 9

Practice 9-1
1. No; the triangles are not the same size. 2. Yes; the ovals are the same shape and size. 3a. \( \angle C' \) and \( \angle F' \) 3b. \( CD \) and \( C'D', D'E' \) and \( D'E', EF' \) and \( E'F', CF' \) and \( C'F' \)
4. \((x, y) \rightarrow (x - 2, y - 4)\) 5. \((x, y) \rightarrow (x + 4, y - 2)\) 6. \((x, y) \rightarrow (x + 2, y + 2)\) 7. \((0, -1, 4)\) 8. \((0, -1, 4)\) 9. \((x + 3, y + 3)\) 10. \((0, -1, 4)\) 11. \((0, -3, -1)\) 11b. \((0, 8), N'(-5, 2), O'(2, 3)\)

Guided Problem Solving 9-1
1. the four vertices of a preimage and one of the vertices of the image 2. Graph the image and preimage. 3. \((4, 2)\) and \((0, 0)\) 4. \(x = 4, y = 2, x + a = 0, y + b = 0\) 5. \(a = -4; b = -2; (x, y) \rightarrow (x - 4, y - 2)\) 6. \(A'(-1, 4), B'(1, 3), D'(-2, 1)\)

Practice 9-2
1. \((-3, -2)\) 2. \((-2, -3)\) 3. \((-1, -4)\) 4. \((4, -2)\) 5. \((4, -1)\) 6. \((3, -4)\) 7a.

Guided Problem Solving 9-2
1. a point at the origin, and two reflection lines 2. A reflection is an isometry in which a figure and its image have opposite orientations. 3. the image after two successive reflections

Practice 9-2

Guided Problem Solving 9-2
4. 5. 6. 7. \((0, -3)\) 8. Yes. The x-coordinate remains 0 throughout. 9. \(O'(0, 0)\) and \(O'(0, 6)\)
Guided Problem Solving 9-3
1. The coordinates of point $A$, and three rotation transformations. It is assumed that the rotations are counterclockwise.  
2. Parallelogram, rhombus, square 
3. slope of $\overline{AB} = \frac{5}{2}$; slope of $\overline{OC} = \frac{2}{5}$; slope of $\overline{OD} = \frac{-5}{2}$; the slopes of perpendicular line segments are negative reciprocals.  
4. square  
5. yes
6. $B(2, 7), C(7, -2), D(-2, -7)$

Practice 9-4
1. The helmet has reflectional symmetry.  
2. The teapot has reflectional symmetry.  
3. The hat has both rotational and reflectional symmetry.
4. line symmetry and 72° rotational symmetry
5. line symmetry and 90° rotational symmetry
6. line symmetry and 45° rotational symmetry
7. line symmetry and 45° rotational symmetry
8. 180° rotational symmetry
9. line symmetry
10. line symmetry and 45° rotational symmetry
11. 
12. 
13. COOK
14. H O X

Guided Problem Solving 9-4
1. the coordinates of one vertex of a figure that is symmetric about the y-axis
2. Line symmetry is the type of symmetry for which there is a reflection that maps a figure onto itself.
3. the coordinates of another vertex of the figure
4. images (and preimages)
5. reflection across the y-axis
6. \((-3, 4)\) 7. Yes 8. \((-6, 7)\)

Practice 9-5
1. \(\frac{5}{2}\) 2. \(\frac{1}{2}\) 3. 2 4. yes 5. no 6. no

Guided Problem Solving 9-5
1. A description of a square projected onto a screen by an overhead projector, including the square’s area and the scale factor in relation to the square on the transparency.
2. The scale factor of a dilation is the number that describes the size change from an original figure to its image.
3. the area of the square on the transparency
4. smaller; The scale factor 16 > 1, so the dilation is an enlargement.
5. \(\frac{1}{16}\) 6. \(\frac{1}{256}\)

Practice 9-6
1. I. D II. C III. B IV. A
2. \(\ell \parallel m\)
3. \(\ell \parallel m\)
4. \(\ell \parallel m\)
5. \(\ell \parallel m\)
6. \(\ell \parallel m\)
7. reflection 8. rotation 9. glide reflection 10. translation

Guided Problem Solving 9-6
1. assorted triangles and a set of coordinate axes
2. a transformation
3. the transformation that maps one