Patterns and Inductive Reasoning

What You’ll Learn
- To use inductive reasoning to make conjectures

...And Why
To predict future sales for a skateboard business, as in Example 4

Check Skills You’ll Need
Here is a list of the counting numbers: 1, 2, 3, 4, 5, . . .
Some are even and some are odd.
1. Make a list of the positive even numbers. 2, 4, 6, 8, 10, . . .
2. Make a list of the positive odd numbers. 1, 3, 5, 7, 9, . . .
3. Copy and extend this list to show the first 10 perfect squares.
   \[1^2 = 1, \ 2^2 = 4, \ 3^2 = 9, \ 4^2 = 16, \ldots\]
4. Which do you think describes the square of any odd number?
   It is odd. It is even. It is odd.

GO for Help
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New Vocabulary
- inductive reasoning
- conjecture
- counterexample

1-1 Using Inductive Reasoning

Inductive reasoning is reasoning that is based on patterns you observe. If you observe a pattern in a sequence, you can use inductive reasoning to tell what the next terms in the sequence will be.

Example 1 Finding and Using a Pattern
Find a pattern for each sequence. Use the pattern to show the next two terms in the sequence.

a. 3, 6, 12, 24, . . .
   \[3, 6, 12, 24 \quad \times 2 \times 2 \times 2\]
   Each term is twice the preceding term. The next two terms are \(2 \times 24 = 48\) and \(2 \times 48 = 96\).

b. \[
   \begin{array}{c}
   \text{Each circle has one more segment through the center to form equal parts. The next two figures:}
   \end{array}
   \]

Quick Check
Write the next two terms in each sequence.

a. 1, 2, 4, 7, 11, 16, 22, . . .  29, 37
b. Monday, Tuesday, Wednesday, . . . Thursday, Friday Answers may vary. Sample:
   \[
   \begin{array}{c}
   \text{Learning style: visual}
   \end{array}
   \]
A conclusion you reach using inductive reasoning is called a **conjecture**.

### Using Inductive Reasoning

Make a conjecture about the sum of the first 30 odd numbers.

Find the first few sums. Notice that each sum is a perfect square.

- \(1 = 1^2\)
- \(1 + 3 = 4 = 2^2\)
- \(1 + 3 + 5 = 9 = 3^2\)
- \(1 + 3 + 5 + 7 = 16 = 4^2\)

Using inductive reasoning, you can conclude that the sum of the first 30 odd numbers is \(30^2\), or 900.

**Quick Check**

Make a conjecture about the sum of the first 35 odd numbers. Use your calculator to verify your conjecture. **The sum of the first 35 odd numbers is 35^2, or 1225.**

Not all conjectures turn out to be true. You can prove that a conjecture is false by finding one counterexample. A **counterexample** to a conjecture is an example for which the conjecture is incorrect.

### Finding a Counterexample

Find a counterexample for each conjecture.

- **a.** The square of any number is greater than the original number.
  - The number 1 is a counterexample because \(1^2 < 1\).
- **b.** You can connect any three points to form a triangle.
  - If the three points lie on a line, you cannot form a triangle.

- **c.** Any number and its absolute value are opposites.
  - The conjecture is true for negative numbers, but not positive numbers.
  - 8 is a counterexample because 8 and \(8\) are not opposites.

**Quick Check**

Alana makes a conjecture about slicing pizza. She says that if you use only straight cuts, the number of pieces will be twice the number of cuts.

Draw a counterexample that shows you can make 7 pieces using 3 cuts. **See left.**

### 2. Teach

#### Guided Instruction

**Teaching Tip**

Point out that the number that is squared equals the number of terms that are added.

**Additional Examples**

1. Find a pattern for the sequence. Use the pattern to show the next two terms in the sequence.
   - \(384, 192, 96, 48, \ldots\) Each term is half the preceding term; 24, 12.

2. Make a conjecture about the sum of the cubes of the first 25 counting numbers. **The sum equals \((1 + 2 + 3 + \ldots + 25)^2.**

3. Find a counterexample for each conjecture.
   - **a.** A number is always greater than its reciprocal.
     - Sample: 1 is not greater than \(\frac{1}{1}; \frac{1}{2}\) and \(-3\) are also counterexamples.
   - **b.** If a number is divisible by 5, then it is divisible by 10.
     - Sample: 25 is divisible by 5 but not by 10.
   - **c.** The price of overnight shipping was $8.00 in 2000, $9.50 in 2001, and $11.00 in 2002. Make a conjecture about the price in 2003. **Sample: The price will be $12.50.**

**Resources**

- Daily Notetaking Guide 1-1 L3
- Daily Notetaking Guide 1-1—Adapted Instruction L1

#### Closure

Explain how you can use a conjecture to help solve a problem. **Sample: A conjecture can be tested to see whether it is a solution.**

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**Advanced Learners**

Have students explore the pattern in Example 2 geometrically by placing 3, then 5, then 7 squares on the top and right sides of the previous square.

**English Language Learners**

Exercises 42-46 rely solely on visual clues. Use these exercises to assess ELL students’ ability to use inductive reasoning to continue a pattern.
A skateboard shop finds that sales of small-wheeled skateboards are decreasing by about 3 skateboards over a period of five consecutive months, sales of small-wheeled skateboards decreased. Use inductive reasoning. Make a conjecture about the number of small-wheeled skateboards the shop will sell in June.

The graph shows that sales of small-wheeled skateboards is decreasing by about 3 skateboards each month. By inductive reasoning you might conclude that the shop will sell 42 skateboards in June.

a. Make a conjecture about the number of small-wheeled skateboards the shop will sell in July. Sample: 39 skateboards

b. Critical Thinking How confident would you be in using the graph to make a conjecture about sales in December? Explain. Not confident; December is too far away.

**EXERCISES**

**Practice and Problem Solving**

**A Practice by Example**

Example 1 (page 4)

**Find a pattern for each sequence. Use the pattern to show the next two terms.**

1. 5, 10, 20, 40, ... 80, 160

2. 3, 33, 333, 3333, ...

3. 1, −1, 2, −2, 3, −3, ...

4. 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ..., $\frac{1}{16}$, $\frac{1}{32}$

5. 15, 12, 9, 6, ..., 3, 0

6. 81, 27, 9, 3, ..., $\frac{1}{3}$


8. 1, 2, 6, 24, 120, ...

9. 2, 4, 8, 16, 32, ..., 64, 128

10. 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, ..., $\frac{1}{1024}$, $\frac{1}{32768}$

11. 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, ..., $\frac{1}{32768}$

12. 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, ..., $\frac{1}{8}$

13. George, John, Thomas, James, ...

14. Martha, Abigail, Martha, Dolley, ...

15. George, Thomas, Abe, Alexander, ...

16. Elizabeth, Louisa, Andrew, Ulysses

17. Draw the next figure in each sequence.

Sample: 

18.

**Example 2** (page 5)

**Use the table and inductive reasoning. Make a conjecture about each value.**

19. The sum of the first 6 positive even numbers

20. The sum of the first 30 positive even numbers

21. The sum of the first 100 positive even numbers

22. Use the pattern in Example 2 to make a conjecture about the sum of the first 100 odd numbers.

23. The sum of the first 100 positive even numbers is 100 · 101, or 10,100.

24. The sum of the first 100 odd numbers is $100^2$, or 10,000.

25–28. Answers may vary. Samples are given.

25. $8 + (−5) = 3$ and $3 \geq 8$

26. $\frac{1}{3} + \frac{1}{2} \geq \frac{1}{3}$ and $\frac{1}{3} + \frac{1}{2} \geq \frac{1}{2}$

27. $−6 − (−4) \leq −6$ and $−6 − (−4) \leq −4$

28. $\frac{1}{3} \times \frac{1}{2} = \frac{3}{2}$ and $\frac{3}{2}$ is improper.
Predict the next term in each sequence. Use your calculator to verify your answer.

23. \[12345679 \times 9 = 11111111\]
24. \[1 \times 1 = 1\]
25. \[12345679 \times 18 = 22222222\]
26. \[11 \times 11 = 121\]
27. \[12345679 \times 27 = 33333333\]
28. \[111 \times 111 = 12321\]
29. \[12345679 \times 36 = 44444444\]
30. \[1111 \times 1111 = 12345,321]\n
Find one counterexample to show that each conjecture is false.

25. The sum of two numbers is greater than either number.
26. The product of two positive numbers is greater than either number.
27. The difference of two integers is less than either integer.
28. The quotient of two proper fractions is a proper fraction.

29. Weather
   The speed with which a cricket chirps is affected by the temperature. If you hear 20 cricket chirps in 14 seconds, what is the temperature? \[\frac{75}{11543}^{\circ}F\]

30. Physical Fitness
   Dino works out regularly. When he first started exercising, he could do 10 push-ups. After the first month he could do 14 push-ups. After the second month he could do 19, and after the third month he could do 25. Predict the number of push-ups Dino will be able to do after the fifth month of working out. How confident are you of your prediction? Explain. See left.

Find a pattern for each sequence. Use the pattern to show the next two terms.

31. 1, 3, 7, 13, 21, \ldots
32. 1, 2, 5, 9, \ldots
33. 0.1, 0.01, 0.001, \ldots
34. 2, 6, 21, 22, 66, 67, \ldots
35. 1, 3, 7, 15, 31, \ldots
36. 0, 1, 3, 7, 15, \ldots
37. M, V, E, M, \ldots
38. AL, AK, AZ, AR, \ldots
39. H, He, Li, Be, \ldots
40. Writing
   Choose two of the sequences in Exercises 31–36 and describe the patterns. See margin.

41. Points along the yellow line are equal distances from both sides of the bike trail (Exercise 41).

42. Draw two parallel lines on your paper. Locate four points on the paper, each an equal distance from both lines. Describe the figure you get if you continue to locate points, each an equal distance from both lines. See margin.

43. You would get points on a third line between and parallel to the first two lines.

44. 45. Multiple Choice
   Find the perimeter when 100 triangles are put together in the pattern shown. Assume that all triangle sides are 1 cm long. B
   \[A \quad 100 \text{ cm} \quad B \quad 102 \text{ cm} \quad C \quad 202 \text{ cm} \quad D \quad 300 \text{ cm}\]

40. Answers may vary. Sample: In Exercise 31, each number increases by increasing multiples of 2. In Exercise 33, to get the next term, divide by 10.
47. **Math in the Media** Read this excerpt from a news article.

Top female runners have been improving about twice as quickly as the fastest men, a new study says. If this pattern continues, women may soon outrun men in competition!

The study is based on world records collected at 10-year intervals, starting in 1905 for men and in the 1920s for women. If the trend continues, the top female and male runners in races ranging from 200 m to 1500 m might attain the same speeds sometime between 2015 and 2055.

Women’s marathon records date from 1955 but their rapid fall suggests that the women’s record will equal that of men even more quickly.

**a.** What conclusion was reached in the study? **a–c. See left.**

**b.** How was inductive reasoning used to reach the conclusion?

**c.** Explain why the conclusion that women may soon be outrunning men may be incorrect. For which race is the conclusion most suspect? For what reason?

48. **Communications** The table shows the number of commercial radio stations in the United States for a 50-year period. See back of book.

- Make a line graph of the data. **back of book.**
- Use the graph and inductive reasoning to make a conjecture about the number of radio stations in the United States in the year 2010. **about 12,000 radio stations**
- How confident are you about your conjecture? Explain. **See back of book.**

### Number of Radio Stations

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>2,835</td>
</tr>
<tr>
<td>1960</td>
<td>4,224</td>
</tr>
<tr>
<td>1970</td>
<td>6,519</td>
</tr>
<tr>
<td>1980</td>
<td>7,871</td>
</tr>
<tr>
<td>1990</td>
<td>9,379</td>
</tr>
<tr>
<td>2000</td>
<td>10,577</td>
</tr>
</tbody>
</table>

Source: Federal Communications Commission

49. **Open-Ended** Write two different number-pattern sequences that begin with the same two numbers. **See left.**

47. **Answers may vary.** Samples are given.

- a. **Women may soon outrun men in running competitions.**
- b. The conclusion was based on continuing the trend shown in past records.
- c. The conclusions are based on fairly recent records for women, and those rates of improvement may not continue. The conclusion about the marathon is most suspect because records date only from 1955.

50. **Error Analysis** For each of the past four years, Paulo has grown 2 in. every year. He is now 16 years old and is 5 ft 10 in. tall. He figures that when he is 22 years old he will be 6 ft 10 in. tall. What would you tell Paulo about his conjecture?

51. **Coordinate Geometry** You are given x- and y-coordinates for 14 points.

a. Graph each point. **See margin.**

b. Most of the points fit a pattern. Which points do not fit? **H and I**

c. Describe the figure that fits the pattern. **a circle**

52. **History** Leonardo of Pisa (about 1175–1258), also known as Fibonacci (fee buh NAH chee), was born in Italy and educated in North Africa. He was one of the first Europeans known to use modern numerals instead of Roman numerals. The special sequence 1, 1, 2, 3, 5, 8, 13, . . . is known as the Fibonacci sequence. Find the next three terms of this sequence. **21, 34, 55**

53. **Time Measurement** Leap years have 366 days. **See back of book.**


- b. Of the years 2010, 2020, 2100, and 2400, which do you think will be leap years?

- c. **Research** Find out whether your conjecture for part (a) and your answer for part (b) are correct. How are leap years determined?
54. **History** When he was in the third grade, German mathematician Karl Gauss (1777–1855) took ten seconds to sum the integers from 1 to 100. Now it’s your turn. Find a fast way to sum the integers from 1 to 100; from 1 to \(n\). 

(Hint: Use patterns.) See margin.

55. **Algebra** Write the first six terms of the sequence that starts with 1, and for which the difference between consecutive terms is first 2, and then 3, 4, 5, and 6. 

a. 1, 3, 6, 10, 15, 21

b. Evaluate \(\frac{n^2 + n}{2}\) for \(n = 1, 2, 3, 4, 5, 6\). 

56. The sum of the numbers from 1 to 10 is 55. The sum of the numbers from 11 to 20 is 155. The sum of the numbers from 21 to 30 is 255. Based on this pattern, what is the sum of numbers from 91 to 100?

- A. 855
- B. 955
- C. 1055
- D. 1155

57. Which of the following conjectures is false?

- F. The product of two even numbers is even.
- G. The sum of two even numbers is even.
- H. The product of two odd numbers is odd.
- J. The sum of two odd numbers is odd.

58. a. How many dots would be in each of the next three figures?

- a–b. See left.

b. Write an expression for the number of dots in the \(n\)th figure.

59. a. Describe the pattern. List the next two equations in the pattern.

b. Guess what the product of 181 and 11 is. Test your conjecture.

c. State whether the pattern can continue forever. Explain. a–c. See margin.

60. Measure the sides \(DE\) and \(EF\) to the nearest millimeter. 30 mm; 40 mm

61. Measure each angle of \(\triangle DEF\) to the nearest degree. \(\angle D: 59^\circ; \angle E: 60^\circ; \angle F: 40^\circ\)

62. Draw a triangle that has sides of length 6 cm and 5 cm with a 90° angle between those two sides. Check students’ work.