What You’ll Learn

- To understand basic terms of geometry
- To understand basic postulates of geometry

... And Why

To explain why a photographer uses a tripod, as in Exercise 44.

Check Skills You’ll Need

Algebra Solve each system of equations.
1. \( y = x + 5 \) (1, 6) 2. \( y = 2x - 4 \) (3, 2) 3. \( y = 2x \) (5, 10)

4. Copy the diagram of the four points \( A, B, C, \) and \( D \). Draw as many different lines as you can to connect pairs of points. See margin, p. 17.

New Vocabulary

- point
- space
- line
- collinear points
- plane
- coplanar
- postulate
- axiom

Hands-On Activity: How Many Lines Can You Draw?

Many constellations are named for animals and mythological figures. It takes some imagination to join the points representing the stars to get a recognizable figure such as Leo the Lion.

How many lines can you draw connecting the 10 points in Leo the Lion?

- Make a table and look for a pattern to help you find out.

1. Mark three points on a circle. Now connect the three points with as many (straight) lines as possible. How many lines can you draw? 3 lines

2. Mark four points on another circle. How many lines can you draw to connect the four points? 6 lines

3. Repeat this procedure for five points on a circle and then for six points. How many lines can you draw to connect the points? 10 lines, 15 lines

4. Use inductive reasoning to tell how many lines you can draw to connect the ten points of the constellation Leo the Lion. 45 lines

In geometry, some words such as point, line, and plane are undefined. In order to define these words you need to use words that need further defining. It is important however, to have general descriptions of their meanings.
You can think of a **point** as a location. A point has no size. It is represented by a small dot and is named by a capital letter. A geometric figure is a set of points. **Space** is defined as the set of all points.

You can think of a **line** as a series of points that extends in two opposite directions without end. You can name a line by any two points on the line, such as \( \overline{AB} \) (read “line \( AB \)”). Another way to name a line is with a single lowercase letter, such as line \( t \) (see above). Points that lie on the same line are **collinear points**.

### Example 1: Identifying Collinear Points

**a.** Are points \( E, F, \) and \( C \) collinear?

If so, name the line on which they lie.

Points \( E, F, \) and \( C \) are collinear.

They lie on line \( m \).

**b.** Are points \( E, F, \) and \( D \) collinear?

If so, name the line on which they lie.

Points \( E, F, \) and \( D \) are not collinear.

**Quick Check**

1. Are points \( F, P, \) and \( C \) collinear? **no**

2. Name line \( m \) in three other ways. **Answers may vary. Sample: \( EF, FC, CE \)**

**c.** **Critical Thinking** Why do you think arrowheads are used when drawing a line or naming a line such as \( EF \)? Arrowheads are used to show that the line extends in opposite directions without end.

### Example 2: Naming a Plane

Each surface of the ice cube represents part of a plane. Name the plane represented by the front of the ice cube.

You can name the plane represented by the front of the ice cube using at least three noncollinear points in the plane. Some names are plane \( AEF \), plane \( AEB \), and plane \( ABFE \).

**Quick Check**

2. List three different names for the plane represented by the top of the ice cube. **Answers may vary. Sample: \( HEF, HEFG, FGH \)**

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**Advanced Learners**

In Example 4, have students find the number of different planes named by any three vertices of the cube.

**English Language Learners**

The word **noncollinear** in Postulate 1-4 may confuse students. Discuss how the prefix *non-* indicates *not*. So, noncollinear points do *not* lie on a line.
A **postulate** or **axiom** is an accepted statement of fact. You have used some of the following geometry postulates in algebra. For example, you used Postulate 1-1 when you graphed an equation such as $y = -2x + 8$. You plotted two points and then drew the line through those two points.

### Key Concepts

**Postulate 1-1**

Through any two points there is exactly one line.

Line $t$ is the only line that passes through points $A$ and $B$.

In algebra, one way to solve a system of two equations is to graph the two equations. As the graphs of

$$
y = -2x + 8
$$

$$
y = 3x - 7
$$

show, the two lines intersect at a single point, $(3, 2)$. The solution to the system of equations is $(3, 2)$.

This illustrates Postulate 1-2.

**Postulate 1-2**

If two lines intersect, then they intersect in exactly one point.

$\overline{AE}$ and $\overline{BD}$ intersect at $C$.

There is a similar postulate about the intersection of planes.

**Postulate 1-3**

If two planes intersect, then they intersect in exactly one line.

Plane $RST$ and plane $STW$ intersect in $ST$.

When you know two points in the intersection of two planes, Postulates 1-1 and 1-3 tell you that the line through those points is the line of intersection of the planes.
**3. Practice**

**Assignment Guide**

- **A B** 1-16, 46-60, 64, 68-73
- **A B** 17-45, 61-63, 65-67
- **C Challenge** 74-79

**Test Prep** 80-84
**Mixed Review** 85-90

**Homework Quick Check**

To check students’ understanding of key skills and concepts, go over Exercises 5, 18, 50, 62, 66.

**Error Prevention!**

Exercises 9, 10 Check that students use a line symbol and not a segment or ray symbol as they name lines.

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**Key Concepts**

**Postulate 1-4**

Through any three noncollinear points there is exactly one plane.

**EXERCISES**

**Practice and Problem Solving**

For more exercises, see Extra Skill, Word Problem, and Proof Practice.

**A Practice by Example**

**Example 1**

- Are the three points collinear? If so, name the line on which they lie.
  1. A, D, E **no**  
  2. B, C, D **yes**; line n  
  3. B, C, F  
  4. A, E, C **yes**; line m  
  5. F, B, D  
  6. F, A, E **no**  
  7. G, F, C **no**  
  8. A, G, C **yes**; line m  
  9. Name line m in three other ways.  
  10. Name line n in three other ways.  

**Answers may vary. Sample: BF, CD, DF**
Exercises 38–43 These exercises introduce the term noncoplanar. Discuss with students how they can derive its meaning from what they already know about the terms collinear and noncollinear.

Exercises 47–50 Discuss the exercises for which students think that drawing a figure for the description is not possible. Have students explain their reasoning. This is a good way to clarify the ideas that form the basis of a formal proof.

Error Prevention!

Exercises 56–61 These exercises reinforce the vocabulary and postulates in the lesson. Have students work with partners to discuss any unclear terms. Emphasize the mathematical importance of the phrase exactly one in Exercise 57. Also point out in Exercise 61 that two lines mean two distinct lines.

Careers

Exercise 66 Ask: What fact about Earth’s surface complicates the work of a surveyor who uses lines and planes? Earth is a sphere and not flat, so distances are measured along curves.

44. Through any three noncollinear points there is exactly one plane. The ends of the legs of the tripod represent three noncollinear points, so they rest in one plane. Therefore, the tripod won’t wobble.

45. Answers may vary. Sample:

46. not possible

47. not possible

48. A • B

49. not possible

50–53. See back of book.

54. Multiple Choice Which three points are not collinear? C

55. Answers may vary. Sample:
How many planes contain each line and point?

55. Intersecting lines are __ coplanar. **always**
56. Two planes __ intersect in exactly one point. **never**
57. Three points are __ coplanar. **always**
58. A plane containing two points of a line __ contains the entire line. **always**
59. Four points are __ coplanar. **sometimes**
60. Two lines __ meet in more than one point. **never**

61. How many planes contain each line and point?
   a. $EF$ and point $G$ __
   b. $FH$ and point $E$ __
   c. $GF$ and point $P$ __
   d. $EP$ and point $G$ __
   e. **Make a Conjecture** What do you think is true of a line and a point not on the line?
   A line and a point not on the line are always coplanar.

In Exercise 62 and 63, sketch a figure for the given information. Then name the postulate that your figure illustrates. 62–63. See margin, pp. 20–21.

62. The noncollinear points $A$, $B$, and $C$ are all contained in plane $N$.
63. Planes $LNQ$ and $MVK$ intersect in $NM$.

64. **Optical Illusions** The diagram (right) is an optical illusion. Which three points are collinear: $A$, $B$, and $C$ or $A$, $B$, and $D$? Are you sure? Use a straightedge to check your answer. $A$, $B$, and $D$

**Writing** Use postulates to explain each situation. 65–67. See margin.

65. A land surveyor can always find a straight line from the point where she stands to any other point she can see.
66. A carpenter knows that a line can represent the intersection of two flat walls.
67. A furniture maker knows that a three-legged table is always steady, but a four-legged table will sometimes wobble.


73. **Coordinate Geometry** Graph the points and state whether they are collinear.

68. $(1, 1), (4, 4), (-3, -3)$
69. $(2, 4), (4, 6), (0, 2)$
70. $(0, 0), (-5, 1), (6, -2)$
71. $(0, 0), (8, 10), (4, 6)$
72. $(0, 0), (0, 3), (0, -10)$
73. $(-2, -6), (1, -2), (4, 1)$

74. How many planes contain the same three collinear points? Explain. **See left.**

75. **Navigation** Rescue teams use Postulates 1-1 and 1-2 to determine the location of a distress signal. In the diagram, a ship at point $A$ receives a signal from the northeast. A ship at point $B$ receives the same signal from due west. Trace the diagram and find the location of the distress signal. Explain how the two postulates help locate the distress signal. **See margin.**

65. **Post. 1-1:** Through any two points there is exactly one line.
66. **Post. 1-3:** If two planes intersect, then they intersect in exactly one line.
67. **The end of one leg might not be coplanar with the ends of the other three legs. (Post. 1-4)**
73. **no**

75. By Post. 1-1, points $D$ and $B$ determine a line and points $A$ and $D$ determine a line. The distress signal is on both lines and, by Post. 1-2, there can be only one distress signal.
76. **Open-Ended** Suppose two points are in plane $P$. Explain why it makes sense that the line containing the points would be in the same plane. **See margin.**

b. Suppose two lines intersect. How many planes do you think contain both lines? You may use the diagram and your answer in part (a) to explain your answer. **See margin.**

### Probability

Points are picked at random from $A$, $B$, $C$, and $D$, which are arranged as shown. Find the probability that the indicated number of points meet the given condition.

77. 2 points, collinear $\frac{1}{4}$

78. 3 points, collinear $\frac{1}{4}$

79. 3 points, coplanar $1$

80. In the figure at the right, which points are collinear with $C$ and $H$? **A**

A. $B$, $F$

B. $E$, $F$, $G$

C. $A$, $D$, $G$, $I$

D. $A$, $D$, $E$, $H$

81. A solid chunk of cheese is to be cut into 4 pieces. What is the least number of slices needed? **J**

F. 5

G. 4

H. 3

J. 2

82. Ronald is making a table. What is the least number of legs that the table should have so that it will not wobble? **B**

A. 4

B. 3

C. 2

D. 1

83. At most, how many lines can contain pairs of the points $P$, $Q$, and $R$? **H**

F. 1

G. 2

H. 3

J. 4

84. Use the figure at the right. **a-b. See margin.**

a. Name all the planes that form the figure.

b. Name all the lines that intersect at $D$. **Exercise 84**

### Mixed Review

**Lesson 1-2** Make an orthographic drawing for each figure. **85–87. See margin.**

85.

86.

87.

### Skills Handbook

Simplify each ratio.

88. $30 \, \text{to} \, 12$ $\frac{5}{2}$

89. $\frac{\pi r^2}{2rt}$ $\frac{r}{2}$

90. $\frac{5}{\pi} \, : \, \frac{5}{7}$ $\frac{1}{3}$

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