Lesson 1-4
Segments, Rays, Parallel Lines and Planes

What You’ll Learn
• To identify segments and rays
• To recognize parallel lines

. . . And Why
To identify compass directions that can be represented by opposite rays, as in Exercise 36

Check Skills You’ll Need
Judging by appearances, will the lines intersect?
1. no 2. yes 3. no

Name the plane represented by each surface of the box.
4. the bottom NMR 5. the top QPL 
6. the front NKL 7. the back PQR 
8. the left side PKN 9. the right side LQR

New Vocabulary
• segment • ray • opposite rays • parallel lines
• skew lines • parallel planes

Identifying Segments and Rays
Many geometric figures, such as squares and angles, are formed by parts of lines called segments or rays. A segment is the part of a line consisting of two endpoints and all points between them.

A ray is the part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.

Opposite rays are two collinear rays with the same endpoint. Opposite rays always form a line.

Real-World Connection
A sunbeam models a ray. The sun is its endpoint.

EXAMPLE
Naming Segments and Rays
Name the segments and rays in the figure at the right.
• The three segments are LP, PQ, and LQ.
• The four rays are LP or LP, PQ or QP, PL or LP, and QL or LP.

Critical Thinking
LP and LP form a line. Are they opposite rays? Explain.
No, they do not have the same endpoint.

Check Skills You’ll Need
For intervention, direct students to:
Basic Postulates of Geometry
Lesson 1-3: Examples 3, 4
Extra Skills, Word Problems, Proof Practice, Ch. 1

Bell Ringer Practice
For intervention, direct students to:
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Differentiated Instruction
Special Needs
Draw line AB on the board and ask: Are AB and BA opposite rays? Why or why not? No, AB and BA are not opposite rays because they do not have only one endpoint in common, they share many points.

Below Level
Remind students that the different notations for line, line segment, and ray readily identify and distinguish them.

learning style: visual
learning style: verbal
Lines that do not intersect may or may not be coplanar.

**Parallel lines** are coplanar lines that do not intersect. **Skew lines** are noncoplanar; therefore, they are not parallel and do not intersect.

Segments or rays are parallel if they lie in parallel lines. They are skew if they lie in skew lines. $AB$ and $CG$ are skew because $AB$ and $CG$ are skew.

### Example 1

#### Identifying Parallel and Skew Segments

**a.** Name all labeled segments that are parallel to $\overrightarrow{BC}$.

- $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{BA}$, $\overrightarrow{BC}$

**b.** Name all labeled segments that are skew to $\overrightarrow{DC}$.

- $\overrightarrow{NJ}$, $\overrightarrow{GJ}$, and $\overrightarrow{III}$

### Example 2

#### Recognizing Parallel Figures

- Parallel planes are planes that do not intersect. A line and a plane that do not intersect are also parallel.

- $ABCD \parallel GHJI$.

- $ABCD \parallel GHI$.

### Differentiated Instruction

**Advanced Learners**

Have students justify the statement, “Skew lines are noncoplanar; therefore they are not parallel and do not intersect.” They may need to reason indirectly.

**English Language Learners**

Use Exercises 25-33 to reinforce the meaning of the new vocabulary in the lesson as well as the terms **always**, **sometimes**, and **never** in the context of mathematical reasoning.
Identifying Parallel Planes

Use the diagram at the right to name the figures.

a. two pairs of parallel planes
   - plane $ABHG \parallel plane DCIJ$
   - plane $ADJ \parallel plane BCI$

b. a line that is parallel to plane $GHIJ$
   - $\overline{AB}$ is parallel to $GHIJ$.

Name the figures.

a. three pairs of parallel planes
   - $\overline{PSWT} \parallel \overline{RQVU}$
   - $\overline{PRUT} \parallel \overline{SQVW}$
   - $\overline{PSQR} \parallel \overline{TWVU}$

Answers may vary. Sample: $\overline{PS}$

Use the figure from Example 3. Name all segments that are parallel to $\overline{AD}$. Name all that are skew to $\overline{AD}$.

parallel: $\overline{GJ}, \overline{HI}, \overline{BC}$
skew: $\overline{GH}, \overline{JI}, \overline{BH}, \overline{CI}$

Identify a pair of parallel planes in your classroom.
Sample: floor and ceiling

Resources
- Daily Notetaking Guide 1-4
- Daily Notetaking Guide 1-4—Adapted Instruction

Closure
How are parallel and skew lines alike? How are they different?
Both parallel and skew lines never intersect; parallel lines are coplanar, whereas skew lines are not.
In Exercises 16–23, describe the statement as true or false. If false, explain.

16. \( \overrightarrow{CB} \parallel \overrightarrow{HG} \) \( \text{true} \)
17. \( \overrightarrow{ED} \parallel \overrightarrow{HG} \) False; they are skew.
18. plane \( AED \parallel \) plane \( FGH \) \( \text{true} \)
19. plane \( ABH \parallel \) plane \( CDF \) False; they intersect above \( CG \).
20. \( AB \) and \( HG \) are skew lines. \( \text{true} \)
21. \( AE \) and \( BC \) are skew lines. \( \text{See left.} \)
22. \( \overrightarrow{CG} \) and \( \overrightarrow{AI} \) are skew lines. False; they are ||.
23. \( \overrightarrow{CF} \) and \( \overrightarrow{AJ} \) are skew lines. False; they are \( \parallel \).
24. The following steps show how to draw planes \( A \) and \( B \) intersecting in \( FG \).

Step 1  
Step 2  
Step 3  

Use similar steps to draw plane \( DFE \) and plane \( DEF \) intersecting in \( DF \). See margin.

Complete Exercises 25–33 with always, sometimes, or never to make a true statement.

25. Two parallel lines are \( ? \) coplanar. always
26. Two skew lines are \( ? \) coplanar. never
27. Two opposite rays \( ? \) form a line. always
28. \( \overrightarrow{TQ} \) and \( \overrightarrow{TJ} \) are \( ? \) the same line. never
29. \( \overrightarrow{GH} \) and \( \overrightarrow{HG} \) are \( ? \) the same ray. sometimes
30. \( \overrightarrow{JK} \) and \( \overrightarrow{LJ} \) are \( ? \) the same ray. always
31. Two planes that do not intersect are \( ? \) parallel. always
32. Two lines that lie in parallel planes are \( ? \) parallel. sometimes
33. Two lines in intersecting planes are \( ? \) skew. sometimes
34. Multiple Choice \( \overrightarrow{FG} \) has endpoints \( F(-3, 3) \) and \( G(3, 1) \). Which point is also on \( \overrightarrow{FG} \)?
   \( C \) \( (-6, 4) \) \( \text{© Pearson Education, Inc. All rights reserved.} \)

35. Coordinate Geometry \( \overrightarrow{AB} \) has endpoint \( A(2, 3) \) and contains \( B(4, 6) \).
   Give possible coordinates for point \( C \) so that \( \overrightarrow{AB} \) and \( \overrightarrow{AC} \) are opposite rays.
   Graph your answer.
   Answers may vary. Sample: \( (0, 0) \); check students' graphs.

36. Directional Compass On a directional compass, the directions north and south can be represented by opposite rays.
   a. Name two other compass directions that can be represented by opposite rays.
   b. What other pairs of opposite directions, if any, can you find? a–b. See margin.

37. Open-Ended Summarize the three ways in which two lines may be related. Give examples from the real world that illustrate the relationships. See margin.
38. Writing The term *skew* is a Middle English word meaning “to escape.” Explain how this meaning might be appropriate for skew lines. **See left.**

39. Critical Thinking Suppose two parallel planes $A$ and $B$ are each intersected by a third plane $C$.
   a. Make a conjecture about the intersection of planes $A$ and $C$ and the intersection of planes $B$ and $C$. **The lines of intersection are parallel.**
   b. Find examples in your classroom. **See margin.**

40. a. Draw a line. Draw points $E$ and $F$ on the line. How many different segments do points $E$ and $F$ determine? Name the segments. **See margin.**
   b. Draw another line. Draw points $E$, $F$, and $G$ on the line. How many segments do points $E$, $F$, and $G$ determine? Name them. **See margin.**
   c. Continue to draw lines, labeling one more point each time. Make a table showing the number of points and the number of segments determined. Look for and describe a pattern in the data. **See margin.**
   d. Use your pattern to find how many segments are determined if you label 10 points on a line. **45 segments**
   e. If you label $n$ points on a line, how many segments can you name? \(n(n - 1)\)

**Use the figure at the right for Exercises 41 and 42.**

41. Do planes $A$ and $B$ have other lines in common that are parallel to $CD$? Explain. **See margin.**

42. Visualization Are there planes that intersect planes $A$ and $B$ in lines parallel to $CD$? Draw a sketch to support your answer. **See margin.**

**The figure at the right is a pyramid.**

43. Name three lines that intersect at one point. **See left.**

44. What line could be parallel to $PS$? **$QR$**

45. Visualization Consider a plane through $V$ that is parallel to plane $PQRS$. Can a line in that plane be parallel to $SR$? Can it intersect $SR$? Can it be skew to $SR$? Explain each answer.
   - Yes; yes; yes; explanations may vary.

**Use the figure at the right for Exercises 46–49.**

46. How many labeled segments are in the figure? **D. 10**

47. Which ray is opposite $BC$? **H. $BA$**

48. What is another name for $CA$? **B. $AC$**

49. Which figure could be the intersection of two planes? **F. line**

**Multiple Choice**

<table>
<thead>
<tr>
<th>Number of points</th>
<th>Number of segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
</tr>
</tbody>
</table>

Answers may vary.

**Sample: For each “new” point, the number of new segments equals the number of “old” points.**

41. No; two different planes cannot intersect in more than one line.

42. Yes; plane $P$, for example
50. a. Alike: They do not intersect. Different: Parallel lines are coplanar and skew lines lie in different planes.

   b. No; of the 8 other lines shown, 4 intersect \( JM \) and 4 are skew to \( JM \).

   [1] one likeness, one difference

9. Answers may vary. Sample: \( \overline{AE} \) and \( \overline{BC} \)

3. Writing Describe the pattern of each sequence in Exercises 1 and 2. See left.

3. For 1: Add 2.5. For 2: Extend the decimal to one more place with a digit that is 1 more than the one to its left.

4. Points \( A, E, F, \) and \( B \)

5. Points \( D, C, E, \) and \( F \)

6. Points \( H, G, F, \) and \( B \)

7. Points \( A, E, B, \) and \( C \)

8. Name all the segments parallel to \( \overrightarrow{HI} \). \( \overrightarrow{CD}, \overrightarrow{AB}, \overrightarrow{EF} \)

9. Name a pair of skew lines. See left.

10. Draw a net for the figure. See margin.

Lesson 1-3

51–58. Answers may vary. Samples are given. Use the diagram for Exercises 51–58 and name each geometric figure.

51. a line \( 
\overrightarrow{EF} \)

52. a point \( A \)

53. the intersection of \( \overrightarrow{DC} \) and \( \overrightarrow{CG} \) \( C \)

54. two planes that intersect in \( \overrightarrow{EF} \) \( AEF \) and \( \overrightarrow{HEF} \)

55. the plane represented by the top of the box \( \overline{ABH} \)

56. the plane represented by the front of the box \( \overline{EHG} \)

57. the intersection of planes \( \overrightarrow{EFG} \) and \( \overrightarrow{DFG} \)

58. another point in plane \( \overline{CGH} \) \( B \)

Draw the following. 59–61. See margin.

59. \( \overline{TR} \)

60. \( \overline{PQ} \)

61. \( \overline{NV} \)

Lesson 1-1

Find the next two terms in each sequence.

62. \( 1, 1.08, 1.16, 1.32, \ldots \) \( 1.4, 1.48 \)

63. \( -1, -2, -4, -7, -11, -16, \ldots \) \( -22, -29 \)

64. \( AB, BC, CD, DE, EF, \ldots \) \( FG, GH \)

65. \( A, D, G, J, M, \ldots \) \( P, S \)

66. Reasoning Raven conjectured: “If you subtract a number from a given number, the result is always less than the given number.” Is her conjecture true? Explain. No; whenever you subtract a negative number, the answer is greater than the given number. Also, if you subtract 0, the answer stays the same.

Checkpoint Quiz 1

Find the next two terms in each sequence.

1. \( 19, 21.5, 24, 26.5, \ldots \) \( 29, 31.5 \)

2. \( 3.45678, 3.456789 \)

3. For 1: Add 2.5. For 2: Extend the decimal to one more place with a digit that is 1 more than the one to its left.

3. Writing Describe the pattern of each sequence in Exercises 1 and 2. See left.

Use the diagram for Exercises 4–10. In Exercises 4–7, do the points appear to be coplanar? If yes, name the plane. If no, explain. 4–7. See margin.

4. Points \( A, E, F, \) and \( B \)

5. Points \( D, C, E, \) and \( F \)

6. Points \( H, G, F, \) and \( B \)

7. Points \( A, E, B, \) and \( C \)

8. Name all the segments parallel to \( \overrightarrow{HI} \). \( \overrightarrow{CD}, \overrightarrow{AB}, \overrightarrow{EF} \)

9. Name a pair of skew lines. See left.

10. Draw a net for the figure. See margin.