Basic Constructions

What You’ll Learn
• To use a compass and a straightedge to construct congruent segments and congruent angles
• To use a compass and a straightedge to bisect segments and angles

...And Why
To construct the bisector of an angle to illustrate angles of incidence and reflection, as in Exercise 18

Check Skills You’ll Need
In Exercises 1–6, sketch each figure. 1–6. See back of book.
1. \( \overline{CD} \)
2. \( \overline{GH} \)
3. \( \overline{AB} \)
4. line \( m \)
5. acute \( \triangle ABC \)
6. \( \overline{XY} \parallel \overline{ST} \)

7. \( \overline{DE} = 20 \). Point \( C \) is the midpoint of \( \overline{DE} \). Find \( \overline{CE} \).

8. Use a protractor to draw a 60° angle. 8–9. See back of book.
9. Use a protractor to draw a 120° angle.

New Vocabulary
• construction
• straightedge
• compass
• perpendicular lines
• perpendicular bisector
• angle bisector

Constructing Segments and Angles

In a construction you use a straightedge and a compass to draw a geometric figure. A straightedge is a ruler with no markings on it. A compass is a geometric tool used to draw circles and parts of circles called arcs.

Four basic constructions involve constructing congruent segments, congruent angles, and bisectors of segments and angles.

EXAMPLE 1 Constructing Congruent Segments

Construct a segment congruent to a given segment.

Given: \( \overline{AB} \)

Construct: \( \overline{CD} \) so that \( \overline{CD} \cong \overline{AB} \)

Step 1
Draw a ray with endpoint \( C \).

Step 2
Open the compass to the length of \( \overline{AB} \).

Step 3
With the same compass setting, put the compass point on point \( C \). Draw an arc that intersects the ray. Label the point of intersection \( D \).

\( \overline{CD} \cong \overline{AB} \)

EXAMPLE 2 Constructing Congruent Angles

Construct an angle congruent to a given angle.

Given: \( \angle ABC \)

Construct: \( \angle DEG \) so that \( \angle DEG \cong \angle ABC \)

Step 1
Draw a ray with endpoint \( D \).

Step 2
Open the compass to the length of \( \overline{BC} \).

Step 3
With the same compass setting, put the compass point on point \( D \). Draw an arc that intersects the ray. Label the point of intersection \( E \).

\( \angle DEG \cong \angle ABC \)

EXAMPLE 3 Constructing the Perpendicular Bisector

Construct the perpendicular bisector of a segment.

Given: \( \overline{AB} \)

Construct: the perpendicular bisector of \( \overline{AB} \)

Step 1
Draw a segment with endpoints \( A \) and \( B \).

Step 2
Open the compass to the length of \( \overline{AB} \).

Step 3
With the same compass setting, put the compass point on point \( A \). Draw an arc that intersects the ray. Label the point of intersection \( C \).

Step 4
With the same compass setting, put the compass point on point \( B \). Draw an arc that intersects the ray. Label the point of intersection \( D \).

Step 5
Use a straightedge to draw \( \overline{CD} \). Then construct \( \overline{RS} \) so that \( RS = 2XY \).

Math Background

Construction methods are justified by postulates such as Euclid’s Fourth Postulate, that a circle can be drawn with any center and any positive radius, and by, for example, triangle congruency theorems. Compass-and-straightedge constructions provide a hands-on introduction to these postulates and theorems.

More Math Background: p. 2D

Lesson Planning and Resources

See p. 2E for a list of the resources that support this lesson.

Bell Ringer Practice

Check Skills You’ll Need
For intervention, direct students to:

Finding Segment Lengths
Lesson 1-3: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 1

Finding Angle Measures
Lesson 1-4: Examples 4 and 5
Extra Skills, Word Problems, Proof Practice, Ch. 1

Differentiated Instruction

Special Needs L5
For Example 3 on constructing a perpendicular bisector, ask: Why does the opening of the compass need to be greater than half the length of the segment? the arcs must intersect

learning style: verbal

Below Level L2
Demonstrate the construction steps in Examples 1, 2, 3, and 5 by using a large demonstration compass. Then ask student volunteers to demonstrate the constructions.

learning style: visual
Lesson 1-7  Basic Constructions

Constructing Congruent Angles

Construct an angle congruent to a given angle.

Given: \( \angle A \)

Construct: \( \angle S \) so that \( \angle S \cong \angle A \)

Step 1
Draw a ray with endpoint \( S \).

Step 2
With the compass point on point \( A \), draw an arc that intersects the sides of \( \angle A \). Label the points of intersection \( B \) and \( C \).

Step 3
With the same compass setting, put the compass point on point \( S \). Draw an arc and label its point of intersection with the ray as \( R \).

Step 4
Open the compass to the length \( BC \). Keeping the same compass setting, put the compass point on \( R \). Draw an arc to locate point \( T \).

Step 5
Draw \( \overline{ST} \).

\( \angle S \cong \angle A \)

2. Quick Check

Construct \( \angle F \) with \( m\angle F = 2m\angle B \).

Perpendicular Bisectors

Perpendicular lines are two lines that intersect to form right angles. The symbol \( \perp \) means “is perpendicular to.” In the diagram at the right, \( \overline{AB} \perp \overline{CD} \) and \( \overline{CD} \perp \overline{AB} \).

A perpendicular bisector of a segment is a line, segment, or ray that is perpendicular to the segment at its midpoint, thereby bisecting the segment into two congruent segments.

As you will learn in Chapter 5, there is just one line that is the perpendicular bisector of a segment in a given plane. Here is a way to construct the perpendicular bisector.
Chapter 1  Tools of Geometry

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Guided Instruction

Teaching Tip
Point out that the symbol for perpendicular resembles perpendicular lines. Ask: What other symbol resembles what it represents? Sample: the symbol for parallel

3 EXAMPLE  Constructing the Perpendicular Bisector

Construct the perpendicular bisector of a segment.

Given: \( \overline{AB} \)

Construct: \( \overline{XY} \) so that \( \overline{XY} \perp \overline{AB} \) at the midpoint \( M \) of \( \overline{AB} \).

Step 1
Put the compass point on point \( A \) and draw a long arc as shown. Be sure the opening is greater than \( AB \).

Step 2
With the same compass setting, put the compass point on point \( B \) and draw another long arc. Label the points where the two arcs intersect as \( X \) and \( Y \).

Step 3
Draw \( \overline{XY} \). The point of intersection of \( \overline{AB} \) and \( \overline{XY} \) is \( M \), the midpoint of \( \overline{AB} \). So \( \overline{XY} \) is the perpendicular bisector of \( \overline{AB} \).

Real-World Connection

Careers Architects use construction tools to work with their designs.

Resources
- Daily Notetaking Guide 1-7
- Daily Notetaking Guide 1-7—Adapted Instruction

Closure

Write steps in your own words for bisecting an angle and bisecting a segment. Use drawings that show the arcs in each step. Check students' work.

4 EXAMPLE  Finding Angle Measures

Algebra \( \overline{KN} \) bisects \( \angle JKL \) so that \( m\angle KNJ = 5x - 25 \) and \( m\angle NKL = 3x + 5 \). Solve for \( x \) and find \( m\angle KNJ \).

\[
\begin{align*}
m\angle KNJ &= m\angle NKL \\
5x - 25 &= 3x + 5 \\
5x &= 3x + 30 \\
2x &= 30 \\
x &= 15
\end{align*}
\]

\[
m\angle KNJ = 5(15) - 25 = 50
\]

Quick Check
4 Find \( m\angle NKL \) and \( m\angle JKL \). 50; 100

Exercises

1. \( \overline{AB} \)

Quick Check
5. a. \( \overline{AB} \)
**EXAMPLE 5** Constructing the Angle Bisector

Construct the bisector of an angle.

**Given:** \( \angle A \)
**Construct:** \( \overline{AX} \), the bisector of \( \angle A \)

**Step 1**
Put the compass point on vertex \( A \). Draw an arc that intersects the sides of \( \angle A \). Label the points of intersection \( B \) and \( C \).

**Step 2**
Put the compass point on point \( C \) and draw an arc. With the same compass setting, draw an arc using point \( B \). Be sure the arcs intersect. Label the point where the two arcs intersect as \( X \).

**Step 3**
Draw \( \overline{AX} \).

\( \overline{AX} \) is the bisector of \( \angle CAB \).

**Quick Check**

3. Draw obtuse \( \angle XYZ \). Then construct its bisector \( \overline{YP} \). **See margin, p. 46.**
4. Explain how you can use your protractor to check your construction.

**EXERCISES**

For more exercises, see Extra Skill, Word Problem, and Proof Practice.

**Practice and Problem Solving**

**A** Practice by Example

**Example 1**

(page 44)

1. Construct \( \overline{XY} \) congruent to \( \overline{AB} \).
2. Construct \( \overline{VW} \) so that \( VW = 2AB \).
3. Construct \( \overline{DE} \) so that \( DE = TR + PS \).
4. Construct \( \overline{OJ} \) so that \( OJ = TR - PS \).
5. Construct \( \angle D \) so that \( \angle D \equiv \angle C \).
6. Construct \( \angle F \) so that \( m \angle F = 2m \angle C \).
7. Construct the perpendicular bisector of \( \overline{AB} \).
8. Construct the perpendicular bisector of \( \overline{TR} \).

**Example 2**

(page 45)

**Example 3**

(page 46)

**Example 4**

(page 46)

9. **Algebra** \( \overline{GH} \) bisects \( \angle FGI \).
   a. Solve for \( x \) and find \( m \angle FGH \).
   b. Find \( m \angle HGI \).
   c. Find \( m \angle FGI \).

**Exercises**

In Exercises 1–8, draw a diagram similar to the given one. Then do the construction. Check your work with a ruler or a protractor. 1–6. **See margin, pp. 46–47.**

1. Construct \( \overline{XY} \) congruent to \( \overline{AB} \).
2. Construct \( \overline{VW} \) so that \( VW = 2AB \).
3. Construct \( \overline{DE} \) so that \( DE = TR + PS \).
4. Construct \( \overline{OJ} \) so that \( OJ = TR - PS \).
5. Construct \( \angle D \) so that \( \angle D \equiv \angle C \).
6. Construct \( \angle F \) so that \( m \angle F = 2m \angle C \).
7. Construct the perpendicular bisector of \( \overline{AB} \).
8. Construct the perpendicular bisector of \( \overline{TR} \).

**Quick Check**

3. Draw obtuse \( \angle XYZ \). Then construct its bisector \( \overline{YP} \). **See margin, p. 46.**
4. Explain how you can use your protractor to check your construction.

**EXERCISES**

For more exercises, see Extra Skill, Word Problem, and Proof Practice.

**Practice and Problem Solving**

**A** Practice by Example

**Example 1**

(page 44)

1. Construct \( \overline{XY} \) congruent to \( \overline{AB} \).
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5. Construct \( \angle D \) so that \( \angle D \equiv \angle C \).
6. Construct \( \angle F \) so that \( m \angle F = 2m \angle C \).
7. Construct the perpendicular bisector of \( \overline{AB} \).
8. Construct the perpendicular bisector of \( \overline{TR} \).

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   a. Solve for \( x \) and find \( m \angle FGH \).
   b. Find \( m \angle HGI \).
   c. Find \( m \angle FGI \).

**Exercises**

In Exercises 1–8, draw a diagram similar to the given one. Then do the construction. Check your work with a ruler or a protractor. 1–6. **See margin, pp. 46–47.**

1. Construct \( \overline{XY} \) congruent to \( \overline{AB} \).
2. Construct \( \overline{VW} \) so that \( VW = 2AB \).
3. Construct \( \overline{DE} \) so that \( DE = TR + PS \).
4. Construct \( \overline{OJ} \) so that \( OJ = TR - PS \).
5. Construct \( \angle D \) so that \( \angle D \equiv \angle C \).
6. Construct \( \angle F \) so that \( m \angle F = 2m \angle C \).
7. Construct the perpendicular bisector of \( \overline{AB} \).
8. Construct the perpendicular bisector of \( \overline{TR} \).

9. **Algebra** \( \overline{GH} \) bisects \( \angle FGI \).
   a. Solve for \( x \) and find \( m \angle FGH \).
   b. Find \( m \angle HGI \).
   c. Find \( m \angle FGI \).

**Homework Quick Check**

To check students’ understanding of key skills and concepts, go over Exercises 4, 14, 18, 26, 28.

**Technology Tip**

Have students investigate what software can model compass and straightedge constructions. Some programs use a mouse and pointer to model the action of compass, straightedge, and pencil.

Exercises 2–4 If necessary, discuss ways that students can copy the segment lengths.
Chapter 1

Tools of Geometry

Visual Learners
Exercises 10–12 Have students draw each angle and its bisector before beginning to solve the exercise.

Connection to Algebra
Exercise 25 This exercise is important to assign. Discuss why both Lani and Denyse are correct.

Exercise 34 Some students may think the answer choices are the same. Encourage them to read the answer choices carefully looking for how each choices differs.

Example 5 (page 47)

Apply Your Skills

Algebra For Exercises 10–12, \( \overline{BX} \) bisects \( \angle ABC \). Solve for \( x \) and find \( m \angle ABC \).

10. \( m \angle ABX = 5x, m \angle XBC = 3x + 10 \); 5; 50
11. \( m \angle ABC = 4x - 12, m \angle ABX = 24 \); 15; 48
12. \( m \angle ABX = 4x - 16, m \angle CBX = 2x + 6 \); 11; 56

13. Draw acute \( \angle PQR \). Then construct its bisector. See margin.
14. Draw right \( \angle TUV \). Then construct its bisector. See margin.
15. Use your protractor and draw \( W \) with \( m \angle W = 120 \). Construct \( \angle Z \cong \angle W \). Then construct the bisector of \( \angle Z \). See margin.

Sketch the figure described. Explain how to construct it. Then do the construction.

16. \( \overline{XY} \perp \overline{YZ} \); 16–17. See margin.
17. \( \overline{NT} \) bisecting right \( \angle PSQ \)

18. Optics A beam of light and a mirror can be used to study the behavior of light. Light that strikes the mirror is reflected so that the angle of reflection and the angle of incidence are congruent. In the diagram, \( \overline{BC} \) is perpendicular to the mirror, and \( \angle ABC \) has a measure of 41°.

a. Name the angle of reflection and find its measure. \( \angle CBD; 41 \)
   
b. Find \( m \angle ABD \). 82
   
c. Find \( m \angle ABE \) and \( m \angle DBF \). 49; 49


a. Draw a mirror and a light beam striking the mirror and reflecting from it.
   
b. Construct the bisector of the angle formed by the incoming and reflected light beams. Label the angles of incidence and reflection.

20. Open-Ended Snoopy can draw squares with his compass. You can only draw circles. You can, however, construct a square. Explain how to do this. Use sketches if needed. Then do the construction. See back of book.

21. Answer these questions about a segment in a plane. Explain each answer.
   
a. How many midpoints does the segment have? See back of book.
   
b. How many bisectors does it have? How many lines in the plane are its perpendicular bisectors? b-c. See left.
   
c. How many lines in space are its perpendicular bisectors?

For Exercises 22–24, copy \( \angle 1 \) and \( \angle 2 \). 22–24. See back of book.

22. Construct \( \angle B \) so that \( m \angle B = m \angle 1 + m \angle 2 \).
23. Construct \( \angle C \) so that \( m \angle C = m \angle 1 - m \angle 2 \).
24. Construct \( \angle D \) so that \( m \angle D = 2m \angle 2 \).

17. Find a segment on \( \overrightarrow{SQ} \) so that you can construct \( \overrightarrow{SP} \) as its \( \perp \) bisector. Then bisect \( \angle PSQ \).
25. They are both correct. If you mult. each side of Lani’s eq. by 2, the result is Denyse’s eq.

26. Reasoning When $BX$ bisects $\angle ABC$, $\angle ABX \cong \angle CBX$. Lani claims there is always a related equation, $m \angle ABX = \frac{1}{2}m \angle ABC$. Denyse claims the related equation is $2m \angle ABX = m \angle ABC$. Which equation is correct? Explain.

27. Writing Describe how to construct the midpoint of a segment. See back of book.

28. a. Draw a large triangle with three acute angles. Construct the bisectors of the three angles. What appears to be true about the three angle bisectors?

b. Repeat the constructions with a triangle that has one obtuse angle.

c. Make a Conjecture What appears to be true about the three angle bisectors of any triangle? a–c. See back of book.

29. with 4-cm, 4-cm, and 5-cm sides

30. with 2-cm, 5-cm, and 5-cm sides

31. with 2-cm, 2-cm, and 4-cm sides

a. Draw a segment, $XY$. Construct a triangle with sides congruent to $XY$.

b. Measure the angles of the triangle. a–c. See margin.

c. Writing Describe how to construct a 60° angle; a 30° angle.

34. Multiple Choice Which steps best describe how to construct this pattern? A

A. Use a straightedge to draw the line segment and then a compass to draw five half circles.

B. Use a straightedge to draw the line segment and then a compass to draw six half circles.

C. Use a compass to draw five half circles and then a straightedge to join their ends.

D. Use a compass to draw six half circles and then a straightedge to join their ends.

35. a. Use your compass to draw a circle. a–c. See back of book.

Locate three points $A$, $B$, and $C$ on the circle.

b. Construct the perpendicular bisectors of $AB$ and $BC$.

c. Critical Thinking Label the intersection of the two perpendicular bisectors as point $O$. Make a conjecture about point $O$.

36. Study the figures. Complete the definition of a line perpendicular to a plane:

A line is perpendicular to a plane if it is $\perp$ to every line in the plane that $\perp$.

1. $\perp$; the line intersects

1. $\perp$; the line intersects

Test Prep

Multiple Choice

37. What must you do to construct the midpoint of a segment? D

A. Measure half its length. B. Measure twice its length.

C. Construct an angle bisector. D. Construct a perpendicular bisector.

38. They are all 60°.

39. a. Answers may vary. Sample: Mark a pt., $A$. Swing a long arc from $A$. From a pt. $P$ on the arc, swing another arc the same size that intersects the arc at a second pt., $Q$. Draw $\angle PAQ$. To construct a 30° $\perp$, bisect the 60° $\perp$.

40. Critical Thinking

Repeat the constructions with a triangle that has one obtuse angle.

41. 17

42. Find $m \angle DNB$. 88

Alternative Assessment

Have each student construct a right triangle using the methods learned in this lesson. Students should write a set of steps that other students could use to complete the construction.

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 75
- Test-Taking Strategies, p. 70
- Test-Taking Strategies with Transparencies
38. Which of these is the first step in constructing a congruent segment? F
   F. Draw a ray.   G. Draw a line.
   H. Label two points.   J. Measure the segment.

Short Response
   a-b. See margin.

Extended Response
   39. Explain how to do each construction using a compass and a straightedge.
   a. Draw an acute angle. \( \angle ABC \). Construct an angle congruent to \( \angle ABC \).
   b. Construct an angle whose measure is twice that of \( \angle ABC \).
   c. Construct a segment that is 1.25 times as long as a given segment.

40. Explain how to do each construction using a compass and a straightedge.
   a. Divide a segment into two congruent segments. a-c. See margin.
   b. Divide a segment into four congruent segments.
   c. Construct a segment that is 1.25 times as long as a given segment.

Mixed Review

Lesson 1-6
41. Use a protractor to draw a 72° angle. See left.

Lesson 1-5
42. \( \angle DEF \) is a straight angle. \( m \angle DEG = 80 \). Find \( m \angle GEF \). 100
43. \( m \angle TUW = 100 \) and \( m \angle VUW = 80 \). Find possible values of \( m \angle TUW \).
   20 and 180

Lesson 1-4
44. \( \overline{AC} \) 6 45. \( \overline{AD} \) 10
46. \( \overline{CD} \) 4 47. \( \overline{BC} \) 3
   \[ -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 \]
48. Draw \( \overline{RS} \).

Use your drawing from Exercise 48. Answer and explain.
49. Are \( \overline{RS} \) and \( \overline{SR} \) opposite rays? No; they do not have the same endpt.
50. Are \( \overline{RS} \) and \( \overline{SR} \) the same segment? Yes; they both represent a segment with endpts. \( R \) and \( S \).

Geometry at Work

Cabinetmakers not only make cabinets but all types of wooden furniture. The artistry of cabinetmaking can be seen in the beauty and uniqueness of the finest doors, shelves, and tables. The craft is in knowing which types of wood and tools to use, and how to use them.

The carpenter’s square is one of the most useful of the cabinetmaker’s tools. It can be applied to a variety of measuring tasks. The figure shows how to use a carpenter’s square to bisect \( \angle O \).

First, mark equal lengths \( OA \) and \( OC \) on the sides of the angle. Then position the square so that \( BA = BC \) to locate point \( B \). Finally, draw \( OB \). \( OB \) bisects \( \angle O \).