Conditional Statements

You have heard if-then statements such as this one:

If you are not completely satisfied, then your money will be refunded.

Another name for an if-then statement is a conditional. Every conditional has two parts. The part following if is the hypothesis, and the part following then is the conclusion.

Identifying the Hypothesis and the Conclusion

Identify the hypothesis and the conclusion of this conditional statement:

If today is the first day of fall, then the month is September.

Hypothesis: Today is the first day of fall.
Conclusion: The month is September.

You can write many sentences as conditionals.

Writing a Conditional

Write each sentence as a conditional.

a. A rectangle has four right angles.
   If a figure is a rectangle, then it has four right angles.

b. A tiger is an animal.
   If something is a tiger, then it is an animal.

Quick Check

Write each sentence as a conditional.

a. An integer that ends with 0 is divisible by 5.
   If an integer ends with 0, then it is divisible by 5.

b. A square has four congruent sides.
   If a figure is a square, then it has 4 congruent sides.
A conditional can have a truth value of true or false. To show that a conditional is true, show that every time the hypothesis is true, the conclusion is also true. To show that a conditional is false, you need to find only one counterexample for which the hypothesis is true and the conclusion is false.

3 **EXAMPLE** Finding a Counterexample

Show that this conditional is false by finding a counterexample:

If it is February, then there are only 28 days in the month.

To show that this conditional is false, you need to find one counterexample that makes the hypothesis true and the conclusion false.

February in the year 2008 is a counterexample. Because 2008 is a leap year, the month of February has 29 days.

The conditional is false because February 2008 is a counterexample.

3 **Quick Check**

Show that this conditional is false by finding a counterexample:

If the name of a state contains the word New, then the state borders an ocean.

The conditional is false because New Mexico is a counterexample.

You can use a Venn diagram to better understand true conditional statements.

4 **EXAMPLE** Using a Venn Diagram

Draw a Venn diagram to illustrate this conditional:

If you live in Chicago, then you live in Illinois.

The set of things that satisfy the hypothesis lies inside the set of things that satisfy the conclusion.

4 **Quick Check**

Draw a Venn diagram to illustrate this conditional:

If something is a cocker spaniel, then it is a dog.

2 **Converses**

The converse of a conditional switches the hypothesis and the conclusion.

5 **EXAMPLE** Writing the Converse of a Conditional

Write the converse of the following conditional.

**Conditional**

If two lines intersect to form right angles, then they are perpendicular.

**Converse**

If two lines are perpendicular, then they intersect to form right angles.

5 **Quick Check**

Write the converse of the following conditional.

If two lines are not parallel and do not intersect, then they are skew.

If two lines are skew, then they are not parallel and do not intersect.
In Lewis Carroll’s *Reasoning and Proof*.

**Conditional Statements and Converses**

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<th>Statement</th>
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<tr>
<td>Conditional</td>
<td>If an angle is a straight angle, then its measure is 180.</td>
<td>( p \rightarrow q )</td>
<td>If ( p ), then ( q ).</td>
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<tr>
<td>Converse</td>
<td>If the measure of an angle is 180, then it is a straight angle.</td>
<td>( q \rightarrow p )</td>
<td>If ( q ), then ( p ).</td>
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7. Answers may vary. Sample: The statement “I breathe when I sleep” can be rewritten as “If I sleep, then I breathe.” The statement “I sleep when I breathe” can be rewritten as “If I breathe, then I sleep.” The two statements are converses, and do not have the same meaning.
1. Identify the hypothesis and the conclusion in the cartoon. **See left.**

Identify the hypothesis and conclusion of each conditional. **2–6. See margin.**

2. If you want to be fit, then you want to get plenty of exercise.

3. **Algebra** If \( x + 20 = 32 \), then \( x = 12 \).

4. “If you can see the magic in a fairy tale, you can face the future.” — Danielle Steel, novelist

5. “If somebody throws a brick at me, I can catch it and throw it back.” — Harry S. Truman

6. “If you can accept defeat and open your pay envelope without feeling guilty, you’re stealing.” — George Allen, former NFL coach

7. Hypothesis: My fans think that I can do everything I say I can do.
   Conclusion: They’re crazier than I am.

8. Hypothesis: I could paint that flower in a huge scale.
   Conclusion: You could not ignore its beauty.

Write each sentence as a conditional.

9. Glass objects are fragile. **If an object is glass, then it is fragile.**

10. **Algebra** \( 3x - 7 = 14 \) implies that \( 3x = 21 \). If \( 3x - 7 = 14 \), then \( 3x = 21 \).

11. Whole numbers that have 2 as a factor are even. **If a whole number has 2 as a factor, then it is even.**

12. All obtuse angles have measure greater than 90. **See left.**

13. Good weather makes a picnic enjoyable.
   **If the weather is good, then a picnic is enjoyable.**

14. Two skew lines do not lie in the same plane.
   **If two lines are skew, then they do not lie in the same plane.**
Show that each conditional is false by finding a counterexample.

15. If it is not a weekday, then it is Saturday. **Sunday**

16. Odd integers less than 10 are prime. Answers may vary. Sample: **9**

17. If you live in a country that borders the United States, then you live in Canada. **Mexico**

18. If you play a sport with a ball and a bat, then you play baseball. Answers may vary. Sample: **softball**

Draw a Venn diagram to illustrate each statement. 19–22. See margin.


20. If you play the flute, then you are a musician.

21. If an angle has measure 40°, then it is acute.

22. Carrots are vegetables.

Write the converse of each conditional statement.

23. If you eat your vegetables, then you grow. If you grow, then you eat your vegetables.

24. If a triangle is a right triangle, then it has a 90° angle. If a triangle has a 90° angle, then it is a right triangle.

25. If two segments are congruent, then they have the same length. If two segments have the same length, then they are congruent.

26. If you do not work, you do not get paid. If you do not get paid, then you do not work.

Write the converse of each conditional statement. Determine the truth values of the original conditional and its converse. 27–32. See margin pp. 84-85.

27. If you travel from the United States to Kenya, then you have a passport. Converse: If you have a passport, then you travel from the U.S. to Kenya. The original conditional is true and the converse is false.

28. Converse: If the coordinates of a point are positive, then it is in the first quadrant. Both statements are true.

29. Converse: If the chemical formula for a substance is H₂O, then it is water. Both statements are true.
48. If a figure is a square, then it has four congruent angles; true.

49. If a figure has four congruent angles, then it is a square; false; a rectangle is not a square.

50. If a figure has four congruent angles, then it has four congruent angles; false; a rhombus is not a square.

51. If a figure has four congruent angles and four congruent sides, then it is a square; true.

52. Answers may vary. Sample: If you had bought Treadmaster tires, you would not have had a flat tire.

Algebra Write the converse of each statement. If the converse is true, write true; if not true, provide a counterexample.

44. If $x^2 > 0$, then $x > 0$; true.

45. If $x^2 = 4$, then $x = 2$; false; $x = -2$. 

46. If $x^3 < 0$, then $x < 0$; true.

47. Advertising Al sees an ad that states, “You want to look good at the beach this summer. Join GoodFit Health Club.” Al figures, “I am going to join GoodFit Health Club, so that I will look good at the beach.”
   a. Write the statement in the ad as a conditional. a-c. See margin.
   b. Write Al’s statement as a conditional.
   c. Writing Explain why the statement in the ad does not have the same meaning as Al’s statement.

48. Write each statement as a conditional.

38. “We’re half the people; we should be half the Congress.” — Jeanette Rankin, former U.S. Congresswoman, calling for more women in office

If we're half the people, then we should be half the Congress.

39. “A great work is made out of a combination of obedience and liberty.”

— Nadia Boulanger, orchestra conductor and musical mentor

If a work is great, then it is made out of a combination of obedience and liberty.

40. “A problem well stated is a problem half solved.”

— Charles F. Kettering, inventor

If a problem is well stated, then it is half solved.

41. If $x - 3 = 15$, then $x = 18$. See left.

42. If $y$ is negative, then $-y$ is positive. See left.

43. If $x = -6$, then $|x| = 6$. See left.

44. If $x < 0$, then $x^2 > 0$. See left.

45. If $x = 2$, then $x^2 = 4$. See left.

46. If $x < 0$, then $x^3 < 0$.

47. If $x > 0$, then $x^2 > x$. See left.

48-51. See left.

52. Answers may vary. Sample: If you want to look good at the beach this summer, then join GoodFit Health Club.

53. Open-Ended Find an ad in which a conditional is used or implied.

Check students’ work.

Write each postulate as a conditional statement. 54–58. See margin.

54. Two intersecting lines meet in exactly one point.

55. Two intersecting planes meet in exactly one line.

56. Two congruent figures have equal areas.

57. Through any two points there is exactly one line.

58. Through any three noncollinear points there is exactly one plane.
Write a statement beginning with All, Some, or No to match each Venn diagram.

59. All integers that are divisible by 8 are divisible by 2.
60. No squares are triangles. (or No squares are triangles.)
61. Some students are musicians. (or Some musicians are students.)

62. Critical Thinking You can write many statements that begin with All or No as conditionals. Give an example of each. (Hint: See Exercises 59–61.)

63. Let a represent an integer. Consider the five statements r, s, t, u, and v:
- r: a is even.
- s: a is odd.
- t: 2a is even.
- u: 2a is odd.
- v: 2a + 1 is odd.

How many statements of the form p → q can you make from these five statements? Decide whether each of the statements is true or false. See back of book.

Multiple Choice

64. Which is the hypothesis of the following statement? If you can stand up, then you can walk.
A. A  
B. N, M, L, K, 4  
C. u
65. Which is the converse of this statement? If you own a dog, then your pet is furry.
F. You can’t sing, then you can’t go with Sarah.
G. If you can’t go with Sarah, then you can sing.
H. If you can’t sing, then you can go with Sarah.
J. If you can go with Sarah, then you can sing.
66. Which statement has a true converse? A
A. If a vehicle is a car, then it has four wheels.
B. If you go to Asia from the United States, then you cross an ocean.
C. If you own a dog, then your pet is furry.
D. If you can stand up, then you can walk.

Short Response

67. Write the converse of the following statement. Determine its truth value.
If Marta is five years old, then she is too young to vote. See margin.

Mixed Review

Lesson 1-9 Find the perimeter of each rectangle with the given base and height.
68. 6 in., 12 in.  69. 3.5 cm, 7 cm  70. 1.2 yd, 18 in.  71. 11 m, 60 cm  72. Find the area of a circle with diameter 10 in. Leave your answer in terms of π. 25π in.²
Lesson 1-8 Find the distance between the points. Round each answer to the nearest tenth.
73. A(1, 2), B(4, -2)  5  74. M(-5, 1), N(0, 5)  6.4  75. K(0, -6), T(2, 3)  9.2
Lesson 1-1 Find the pattern for each sequence. Use the pattern to show the next two terms.
76. 4, 2, 1, 4, . . . Divide the previous term by 2; 1 4  77. 5, 2, -1, -4, . . . Subtract 3 from the previous term; -7, -10.
78. N, M, L, K, . . . See left.

62. Answers may vary. Sample: All apples are fruits; Conditional: If something is an apple, then it is a fruit. No line segments are rays.
63. [2] If Marta is too young to vote, then she is 5 years old. It is a false statement.
64. [1] one of the two sentences above
65. J
66. D
67. J