### Deductive Reasoning

**What You’ll Learn**
- To use the Law of Detachment
- To use the Law of Syllogism

**... And Why**
To use deductive reasoning to conclude that the Nile River is the longest river in the world, as in Example 5

### Using the Law of Detachment

In Chapter 1 you learned that inductive reasoning is based on observing what has happened and then making a conjecture about what will happen. In this lesson, you will study deductive reasoning.

**Deductive reasoning** (or logical reasoning) is the process of reasoning logically from given statements to a conclusion. If the given statements are true, deductive reasoning produces a true conclusion.

Many people use deductive reasoning in their jobs. A physician diagnosing a patient’s illness uses deductive reasoning. A carpenter uses deductive reasoning to determine what materials are needed at a work site.

**Real-World Connection**

**Careers** An auto mechanic uses deductive reasoning as in Example 1.

**Auto Maintenance** An auto mechanic knows that if a car has a dead battery, the car will not start. A mechanic begins work on a car and finds the battery is dead. What conclusion can she make?

- The mechanic can conclude that the car will not start.

**Critical Thinking** Suppose that a mechanic begins work on a car and finds that the car will not start. Can the mechanic conclude that the car has a dead battery?

**Explain.** No, there could be other things wrong with the car, such as a faulty starter.

In Example 1 the mechanic is using a law of deductive reasoning called the **Law of Detachment.**

### Check Skills You’ll Need

**Write the converse of each statement.** 1–2. See back of book.

- 1. If you don’t sleep enough, then your grades suffer.
- 2. If you want to arrive on time, then you must start early.

**Write each statement as a conditional.**

- 3. Leap years have 366 days.
- 4. Students who do not complete their homework will have lower grades.
- 5. Two lines that are perpendicular meet to form right angles.
- 6. Every sixteen-year-old is a teenager.


### New Vocabulary
- deductive reasoning
- Law of Detachment
- Law of Syllogism

### Math Background

Deductive reasoning is a process of reasoning logically from given facts to a conclusion. The Law of Detachment is found in almost every line of two-column proofs, where \( q \) is the “conclusion” and \( p \to q \) is the “justification” for \( q \). It is \( p \), the given, that is sometimes lost when thinking about each line.

### Lesson Planning and Resources

See p. 78E for a list of the resources that support this lesson.

### Bell Ringer Practice

**Check Skills You’ll Need**
For intervention, direct students to:

**Conditional Statements**
Lesson 2-1: Example 2
Extra Skills, Word Problems, Proof Practice, Ch. 2

**Converses**
Lesson 2-1: Example 5
Extra Skills, Word Problems, Proof Practice, Ch. 2

### Special Needs

**L1** Show students that two distinct criteria must be met in the Law of Detachment (the hypothesis and conditional statement must be true) and in the Law of Syllogism (both statements must be true).

**L2** The wordiness of Examples 4 and 5 is typical of complex arguments. Show students how to substitute letters for hypotheses and conclusions, as in the properties.

**Learning Style:** verbal, visual
**Key Concepts**

**Property** **Law of Detachment**

If a conditional is true and its hypothesis is true, then its conclusion is true.

In symbolic form:

\[ p \rightarrow q \text{ is a true statement and } p \text{ is true, then } q \text{ is true.} \]

**Using the Law of Detachment**

For the given true statements, what can you conclude?

**Given**: If \( M \) is the midpoint of a segment, then it divides the segment into two congruent segments.

\[ M \text{ is the midpoint of } \overline{AB}. \]

You are given that a conditional and its hypothesis are true. By the Law of Detachment, you can conclude that \( M \) divides \( \overline{AB} \) into two congruent segments, or \( AM = MB \).

**Quick Check**

If a baseball player is a pitcher, then that player should not pitch a complete game two days in a row. Vladimir Nuñez is a pitcher. On Monday, he pitches a complete game. What can you conclude? *Answers may vary. Sample: Vladimir Nuñez should not pitch a complete game on Tuesday.*

**Connection to Logic**

The error illustrated is sometimes called the *fallacy of the converse*. Explain that a fallacy is an error in logical thinking. Ask: *Why is this error a “fallacy of the converse”? The fallacy is concluding that the converse is true because the conditional is true.*

**Using the Law of Syllogism**

Another law of deductive reasoning is the Law of Syllogism. The **Law of Syllogism** allows you to state a conclusion from two true conditional statements when the conclusion of one statement is the hypothesis of the other statement.

**Property** **Law of Syllogism**

If \( p \rightarrow q \) and \( q \rightarrow r \) are true statements, then \( p \rightarrow r \) is a true statement.

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**Guided Instruction**

**2. Teaching Tip**

Discuss the use of the word *Given* before the two statements. Point out that in mathematics, the statements following *Given* are considered true. Ask: *What do you call statements that are assumed to be true without proof? axioms, postulates*

**3. Connection to Logic**

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**Advanced Learners**

Use Example 3 as a springboard for students to research other logical fallacies.

**English Language Learners**

The Law of Syllogism uses many words to explain a complex relationship. Discuss how the symbolic form summarizes it clearly and succinctly. Have students explain how the two forms are related.

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**Quick Check**

If a baseball player is a pitcher, then that player should not pitch a complete game two days in a row. Vladimir Nuñez is a pitcher. On Monday, he pitches a complete game. What can you conclude? Answers may vary. Sample: Vladimir Nuñez should not pitch a complete game on Tuesday.

**Quick Check**

If possible, use the Law of Detachment to draw a conclusion. If it is not possible to use this law, explain why.

**Given**: If a road is icy, then driving conditions are hazardous. Driving conditions are hazardous. *Not possible: you do not know that the hypothesis is true.*

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**Additional Examples**

1. A gardener knows that if it rains, the garden will be watered. It is raining. What conclusion can he make? The garden will be watered.

2. For the given statements, what can you conclude?

   **Given**: If \( \triangle A \) is acute, \( m\angle A < 90 \).
   \( \angle A \) is acute. \( m\angle A < 90 \)

3. Does the following argument illustrate the Law of Detachment?

   **Given**: If you make a field goal in basketball, you score two points. Jenna scored two points in basketball. Jenna made a field goal. *No*
CHAPTER 2

If a student wants to go to college, then the student must study hard.

Connection

Use the Law of Syllogism to draw a conclusion from the following

Real-World

If a student gets an A on a final exam, then the student will pass the course.

If a number ends in 6, then it is divisible by 2.

If a number ends in 0, then it is divisible by 10.

Use the Law of Detachment and the Law of Syllogism to draw conclusions.

Reasoning and Proof

- Daily Notetaking Guide 3-3—
- Daily Notetaking Guide 2-3

Resources

- Adapted Instruction

Closure

Use either law from this lesson to draw a conclusion from the two statements. Explain which law you used.

If it rains, then Jan stays inside.
If Jan stays inside, then she does not get wet.

The Law of Syllogism lets you conclude: If it rains, then Jan does not get wet.

EXERCISES

For more exercises, see Extra Skill, Word Problem, and Proof Practice.

Practice and Problem Solving

Practice by Example

Examples 1 and 2 (pages 94, 95)

Use the Law of Detachment to draw a conclusion.

1. If a student gets an A on a final exam, then the student will pass the course.
   Felicia gets an A on the music theory final exam.   Felicia will pass the music theory course.

2. If a student wants to go to college, then the student must study hard.
   Rashid wants to go to the University of North Carolina.
   Rashid must study hard.

14–15. Answers may vary. Samples are given.

14. If an Alaskan mountain is over 20,300 ft high, then it is the highest in the United States.
   Alaska’s Mt. McKinley is the highest in the United States.

15. If you live in Lubbock, then you live in the 28th state to enter the Union.
   Levon lives in the 28th state to enter the Union.

16. Must be true; by (E) and (A), it is breakfast time. Then by (D), Julio drinks juice.
3. If two lines are parallel, then they do not intersect. Line \( e \) is parallel to line \( m \). \textbf{Line \( e \) and line \( m \) do not intersect.}

4. If there is lightning, then it is not safe to be out in the open. Marla sees lightning from the soccer field. \textbf{It is not safe for Marla to be out in the open.}

If possible, \textbf{use the Law of Detachment to draw a conclusion. If not possible, write not possible.}

5. If a figure is a rectangle, then it has two pairs of parallel sides. Figure \( ABCD \) is a rectangle. \textbf{Figure \( ABCD \) has two pairs of parallel sides.}

6. \textbf{Algebra} If \( n \) is a prime number greater than 2, then \( n^2 \) is an odd number. \( 9^2 \) is an odd number. \textbf{not possible}

7. If three points are on the same line, then they are collinear. Points \( X, Y, \) and \( Z \) are on line \( m \). \textbf{Points \( X, Y, \) and \( Z \) are collinear.}

8. If an angle is obtuse, then it is not acute. \( \angle XYZ \) is not obtuse. \textbf{not possible}

9. If you are studying botany, then you are studying biology. \textbf{If a person lives in Omaha, then he or she lives in Nebraska.}

10. If you enjoy yourself, then your time is well spent. \textbf{If you are studying botany, then you are studying biology.}

Use the \textbf{Law of Syllogism} to draw a conclusion. 10-13. \textbf{See left.}

10. \textbf{Zoology} If an animal is a red wolf, then its scientific name is \textit{Canis rufus}. If an animal is named \textit{Canis rufus}, then it is endangered.

11. If two planes intersect, then they intersect in a line. If two planes are not parallel, then they intersect.

12. If you read a good book, then you enjoy yourself. If you enjoy yourself, then your time is well spent.

13. If you are studying botany, then you are studying a science. If you are studying botany, then you are studying biology.

\textbf{Geography} Use the Law of Detachment and the Law of Syllogism to draw conclusions from the following statements. 14-15. \textbf{See margin p. 96.}

14. If a mountain is the highest in Alaska, then it is the highest in the United States. If an Alaskan mountain is over 20,300 ft high, then it is the highest in Alaska. Alaska’s Mount McKinley is 20,320 ft high.

15. If you live in Lubbock, then you live in Texas. If you live in Lubbock, then you live in the 28th state to enter the Union.

For Exercises 16–21, assume that the following statements are true.

A. If Maria drinks juice, then it is breakfast time.

B. If it is lunchtime, then Kira drinks milk and nothing else.

C. If it is mealtime, then Curtis drinks water and nothing else.

D. If it is breakfast time, then Julio drinks juice and nothing else.

E. Maria drinks juice.

Use only the information given above. For each statement, write \textit{must be true}, \textit{may be true}, \textit{or is not true}. Explain your reasoning. 16–21. \textbf{See margin pp. 96-97.}


\textbf{Homework Quick Check} To check students’ understanding of key skills and concepts, go over Exercises 4, 14, 26, 28, 32.

\textbf{Error Prevention!}

Exercise 6 Discuss why the Law of Detachment does not apply to Exercise 6. Ask students to state why a conclusion would not make sense and to provide counterexamples.

Exercise 8 Because this exercise shows a fallacy not seen before in this lesson, students may think it shows valid reasoning. Discuss counterexamples.

\textbf{Alternative Method}

Exercises 10–13 Have students write each statement symbolically after defining which statements the letters \( p, q, \) and \( r \) represent. Then have them write each conclusion \( p \rightarrow r \) in words.
Exercises 16–21 The list of statements assumed to be true may seem daunting to students. Before they begin the exercises, ask: How many of the given statements are conditionals, and how many are simple statements? Four conditionals and one simple statement Point out that the simple statement makes the hypothesis of conditional statement A true.

Exercise 32 Students must use indirect reasoning to conclude that Anita and Beth did not go. Discuss as a class how students reasoned through this exercise. Point out that this type of reasoning becomes increasingly important as students continue studying mathematics.

Math Tip Exercise 33 This exercise establishes the logical rule that when a statement is true, its contrapositive is also true. Symbolically, if \( p \rightarrow q \) is a true statement, then \( \neg q \rightarrow \neg p \) is a true statement.

Real-World Connection
Average ocean temperature in Key West is about 80°F, a good snorkeling temperature.

26. Answers may vary. Sample: If a student wears a hat to school, then the student must take it off indoors. Amy wears a hat to school. Then Amy must take off the hat indoors.

28. No; guys with beards cannot park on Monday.

30. No; there is no parking Tuesday from 6:49 A.M. to 9:11 A.M.

31. Is a person with a blue car allowed to park here on Tuesday at 9:05 A.M.? Yes

32. Reasoning Assume that the following statements are true.
If Anita goes to the concert, Beth will go.
If Beth goes to the concert, Aisha will go.
If Aisha goes to the concert, Ramon will go.
Only two of the four students went to the concert. Who were they? Aisha and Ramon

33. Critical Thinking Consider the following given statements and conclusion.
Given: If an animal is a fish, then it has gills.
A turtle does not have gills.
You conclude: A turtle is not a fish.
This argument does not use the Law of Syllogism or the Law of Detachment, but it does use good deductive reasoning.
a. Draw a Venn diagram to illustrate the given information. See left.
b. Use the Venn diagram to help explain why the argument uses good reasoning. Turtles are not in the circle of animals with gills, so a turtle is not a fish.

36.[2] a. Bert
b. Andrea, Bert, and Carl; if Darla were reading King Lear, all four people would be reading it.

37. [4] a. Harold; Clara and Mark won’t eat sandwiches, so Harold had the sandwich. Since Harold had the sandwich, he also had the milk.
b. Salad; because Mark won’t eat salad or bread, he had the soup. Since he had the soup, he had
the iced tea. We know from part (a) that Harold had the sandwich and milk. Thus Clara had the salad and mineral water.

**Lesson Quiz**

Use the three statements below.

A. If games are canceled, then Maria reads a book.
B. If it snows, then games are canceled.
C. It is snowing.

1. Using only statements A and B, what can you conclude? If it snows, then Maria reads a book.
2. Using only statements B and C, what can you conclude? Games are canceled.
4. Suppose both statement B and "games are canceled" are true. Can you conclude that statement C is true? Explain. No; sample: you cannot apply the Law of Detachment.

**Alternative Assessment**

Have each student follow these two rules to write a logic puzzle.

• The puzzle must contain at least three statements.
• The puzzle must be solvable using the Law of Detachment and the Law of Syllogism.

Have students exchange puzzles and solve. Check students' work.

**Test Prep**

Resources

For additional practice with a variety of test item formats:

• Standardized Test Prep, p. 121
• Test-Taking Strategies, p. 116
• Test-Taking Strategies with Transparencies
1. Identify the hypothesis and the conclusion of this conditional statement:
   If \( x > 5 \), then \( x^2 > 25 \).
   Hypothesis: \( x > 5 \)  Conclusion: \( x^2 > 25 \)

2. Write this statement as a conditional: Roses are beautiful flowers.
   If something is a rose, then it is a beautiful flower.

For Exercises 3 and 4, use this conditional statement:
If an integer ends with 0, then the integer is divisible by 2.

3. Write the converse of the statement.
   If an integer is divisible by 2, then the integer ends with 0.

4. Find a counterexample to show that the converse is not true.
   Answers may vary. Sample: 42 is divisible by 2, but it does not end with 0.

5. Write the two conditionals that make up this biconditional:
   An angle is an acute angle if and only if its measure is between 0 and 90.

5. If an angle is an acute angle, then its measure is between 0 and 90. If an angle's measure is between 0 and 90, then it is an acute angle.

6. Rewrite this definition as a biconditional:
   Points that lie on the same line are collinear.

6. Points are collinear if and only if they lie on the same line.

7. Find a counterexample to show that the following statement is not a good definition:
   A computer is a machine with a keyboard and a memory. Answers may vary. Sample: A graphing calculator has a keyboard and a memory.

8. Use the Law of Detachment or the Law of Syllogism to draw a conclusion from each pair of statements. If not possible, write not possible.
   If a student is on the basketball team, then that student has passing grades.
   Theresa is on the basketball team. Theresa has passing grades.

9. If a student studies geometry, then the student studies mathematics.
   If a student studies mathematics, then the student's mind is expanded.
   If a student studies geometry, then the student's mind is expanded.

10. If you miss the bus, then you will be late for school.
    You are late for school. not possible

A Point in Time

Most people are not detectives, but as a young woman, the English writer Agatha Christie (1890–1976) correctly deduced that many people would like to be. In 1920 she published her first book, a detective novel entitled The Mysterious Affair at Styles in which she introduced the eccentric and ultra-logical Belgian detective Hercule Poirot. In this and in many subsequent novels, Poirot solves mysteries not with guns or car chases but with logical reasoning.

Go Online

For: Information about Agatha Christie
Web Code: aue-2032