**Lesson 1-6**

For Exercises 1–5, use the figure at the right.

1. Name ∠1 in two other ways. ∠AOB, ∠BOA
2. Name the vertex of ∠2. O
3. If ∠1 ≅ ∠2, name the bisector of ∠AOC. OB
4. If m∠AOC = 90 and m∠1 = 45, find m∠2. 45
5. If m∠AOC = 90, name two perpendicular rays. OA and OC

**New Vocabulary**
- Reflexive Property
- Symmetric Property
- Transitive Property

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**Connecting Reasoning in Algebra and Geometry**

In geometry you accept postulates and properties as true. You use deductive reasoning to prove other statements. Some of the properties that you accept as true are the properties of equality from algebra. They are listed below in terms of any numbers a, b, and c.

**Summary**

<table>
<thead>
<tr>
<th>Property</th>
<th>The Distributive Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(b + c) = ab + ac</td>
<td></td>
</tr>
</tbody>
</table>

You also assume that other properties from algebra are true.

**Key Concepts**

- **Addition Property** If a = b, then a + c = b + c.
- **Subtraction Property** If a = b, then a - c = b - c.
- **Multiplication Property** If a = b, then a · c = b · c.
- **Division Property** If a = b and c ≠ 0, then \( \frac{a}{c} = \frac{b}{c} \).
- **Reflexive Property** a = a
- **Symmetric Property** If a = b, then b = a.
- **Transitive Property** If a = b and b = c, then a = c.
- **Substitution Property** If a = b, then b can replace a in any expression.
You use deductive reasoning every time you solve an equation. You can justify each statement that you make with a postulate, a property, or a definition. When you solve problems involving angle measures, you can use the Angle Addition Postulate.

**EXAMPLE 1** Justifying Steps in Solving an Equation

**Algebra** Solve for $x$ and justify each step.

**Given:** $m \angle AOC = 139$

\[
m \angle AOB + m \angle BOC = m \angle AOC
\]
\[
x + 2x + 10 = 139
\]
\[
3x + 10 = 139
\]
\[
3x = 129
\]
\[
x = 43
\]

Fill in each missing reason.

**Given:** $LM$ bisects $\angle KLN$.

\[
LM \text{ bisects } \angle KLN.
\]

\[
m \angle MLN = m \angle MKN
\]
\[
4x = 2x + 40
\]
\[
2x = 40
\]
\[
x = 20
\]

You can use the Segment Addition Postulate to justify statements about lengths of segments.

**EXAMPLE 2** Justifying Steps in Solving an Equation

**Algebra** Solve for $y$ and justify each step.

**Given:** $AC = 21$

\[
AB + BC = AC
\]
\[
2y + (3y - 9) = 21
\]
\[
5y - 9 = 21
\]
\[
5y = 30
\]
\[
y = 6
\]

Find $AB$ and $BC$ by substituting $y = 6$ in the expressions in the diagram above. Check that $AB + BC = 21$. $AB = 12; BC = 9; AB + BC = 12 + 9 = 21$

The Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence. You can use properties of congruence to justify statements.
Name the property of equality or congruence that justifies each statement.

(a) \( \angle K \cong \angle K \)  
   Reflexive Property of Congruence

(b) If \( 2x - 8 = 10 \), then \( 2x = 18 \).  
   Addition Property of Equality

(c) If \( RW \cong TW \) and \( TW \cong PQ \), then \( RW \cong PQ \).  
   Transitive Property of Congruence

(d) If \( m\angle A = m\angle B \), then \( m\angle A = m\angle B \).  
   Symmetric Property of Equality

Name the property of equality or congruence illustrated.

(a) \( XY \cong XY \)  
   Reflexive Prop. of \( \cong \)

(b) If \( m\angle A = 45 \) and \( 45 = m\angle B \), then \( m\angle A = m\angle B \).  
   Transitive or Substitution Prop. of Equality

**EXERCISES**

**Algebra** Fill in the reason that justifies each step.

1. Solve for \( x \).
   \[ m\angle CDE + m\angle EDF = 180 \]
   \[ x + (3x + 20) = 180 \]
   \[ 4x + 20 = 180 \]
   \[ 4x = 160 \]
   \[ x = 40 \]
   \[ a. ? \]
   \[ b. ? \]
   \[ c. ? \]
   \[ d. ? \]
   \[ e. ? \]

2. Solve for \( n \).
   Given: \( XY = 42 \)
   \[ XZ + ZY = XY \]
   \[ 3(n + 4) + 3n = 42 \]
   \[ 3n + 12 + 3n = 42 \]
   \[ 6n + 12 = 42 \]
   \[ 6n = 30 \]
   \[ n = 5 \]
   \[ a. ? \]
   \[ b. ? \]
   \[ c. ? \]
   \[ d. ? \]
   \[ e. ? \]
   \[ f. ? \]

**Additional Examples**

1. Justify each step used to solve \( 5x - 12 = 32 + x \) for \( x \).
   1. \( 5x = 44 + x \)
   2. \( 4x = 44 \)
   3. \( x = 11 \)

2. Suppose points \( A, B, \) and \( C \) are collinear with point \( B \) between points \( A \) and \( C \). Solve for \( x \) if \( AB = 4 + 2x, BC = 15 - x \), and \( AC = 21 \). Justify each step.
   \( AB + BC = AC \)
   \( (4 + 2x) + (15 - x) = 21 \)
   \( (Subst. Prop. of Equality), \)
   \( 19 + x = 21 \)
   \( (Simplify), \)
   \( x = 2 \)
   \( (Subr. Prop. of Equality) \)

3. Name the property that justifies each statement.
   a. If \( x = y \) and \( y + 4 = 3x \), then \( x + 4 = 3x \).  
   Substitution Property of Equality
   b. If \( x + 4 = 3x \), then \( 4 = 2x \).  
   Subtraction Property of Equality
   c. If \( \angle P \cong \angle Q, \angle Q \cong \angle R \), and \( \angle R \cong \angle S \), then \( \angle P \cong \angle S \).  
   Transitive Property of Congruence
   d. If \( \angle A \cong \angle B, \) and \( \angle B \cong \angle C \), then \( \angle A \cong \angle C \).  
   Transitive Property of Congruence

**Closure**

Joy and Hue solve the equation \( 3x = 18 \). Each writes \( x = 6 \). Joy names the Division Property of Equality to justify the step. Hue names the Multiplication Property of Equality to justify the step. Explain why both Joy and Hue are correct.  
**Division by 3 is the same as multiplication by 1/3.**
Give a reason for each step.

   b. Subtr. Prop. of =
   c. Div. Prop. of =

Example 3 (page 105)

Apply Your Skills

Use the given property to complete each statement.

22. Substitution Property

25. Answers may vary. Sample: LR and RL are different ways to name the same segment and ∠CBA and ∠ABC are different ways to name the same ∠.

26. Use what you know about transitive properties to complete the following:

The Transitive Property of Falling Dominos:

If domino A causes domino B to fall, and domino B causes domino C to fall, then domino A causes domino to fall.

See left.
27. **Algebra** Fill in the reason that justifies each step.

Given: \( C \) is the midpoint of \( \overline{AD} \).

\[
\begin{align*}
C &= \text{the midpoint of } \overline{AD} \\
AC &= CD \\
4x &= 2x + 12 \\
2x &= 12 \\
x &= 6
\end{align*}
\]

28. **Algebra** In the figure at the right, \( KM = 35 \).

a. Solve for \( x \). Justify each step. See margin.

b. Find the length of \( KL \). See margin.

29. **Algebra** In the figure at the right, \( m\angle GFI = 128 \).

a. Solve for \( x \). Justify each step. See margin.

b. Find \( m\angle EFI \). See margin.

30. **Algebra** Point \( C \) is on the crease when you fold \( \overline{BD} \) onto \( \overline{BA} \). Give the reason that justifies each step. (Hint: See page 102, Exercises 4 and 5.)

\[
\begin{align*}
\overline{BC} &= \text{bisects } \angle \text{ABD} \\
m\angle ABC &= m\angle CBD \\
6n + 1 &= 4n + 19 \\
2n &= 18 \\
\therefore n &= 9
\end{align*}
\]

31. **Error Analysis** The steps below “show” that \( 1 = 2 \). Find the error.

In the fifth step, each side is divided by \( b - a \).

**Given:** \( a = b \)

**But:** \( b - a = 0 \) and division by 0 is not defined.

**a.** \( a = b \) \( \rightarrow \) Given

**b.** \( ab = b^2 \) \( \rightarrow \) Multiplication Property of Equality

**c.** \( ab - a^2 = b^2 - a^2 \) \( \rightarrow \) Subtraction Property of Equality

**d.** \( a(b - a) = (b + a)(b - a) \) \( \rightarrow \) Distributive Property

**e.** \( a = b + a \) \( \rightarrow \) Division Property of Equality

**f.** \( a = a + a \) \( \rightarrow \) Substitution Property

**g.** \( a = 2a \) \( \rightarrow \) Simplify

**h.** \( 1 = 2 \) \( \rightarrow \) Division Property of Equality

**Relationships:** The relationships “is equal to” and “is congruent to” are reflexive, symmetric, and transitive. In a later chapter, you will see that this is also true for the relationship “is similar to.” Consider the following relationships among people. State whether each relationship is reflexive, symmetric, transitive, or none of these.

**Sample:** The relationship “is younger than” is transitive. If Sue is younger than Fred and Fred is younger than Alana, then Sue is younger than Alana. The relationship “is younger than” is not reflexive because Sue is not younger than herself. It is also not symmetric because if Sue is younger than Fred, Fred is not younger than Sue.

32. has the same birthday as **reflexive**, **symmetric**, **transitive**

33. is taller than **transitive**

34. lives in the same state as **reflexive**, **symmetric**, **transitive**

35. lives in a different state than **symmetric**

36. is the same height as **reflexive**, **symmetric**, **transitive**

37. is a descendant of **transitive**

30a. **By construction**

b. **Def. of \( \triangle \) Bisector**

c. **Subst. Prop.**

d. **Subtr. Prop. of =**

e. **Div. Prop. of =**

31a. For a guide to solving Exercise 28, see p. 109.

31b. **Def. of \( \triangle \) Bisector**

31c. **Subst. Prop.**

31d. **Subtr. Prop. of =**

31e. **Div. Prop. of =**

4. **Assess & Reteach**

**Lesson Quiz**

Name the justification for each statement.

1. \( ab = ab \) **Reflexive Prop. of Eq.**

2. If \( m\angle ABC + 40 = 85 \), then \( m\angle ABC = 45 \). **Subtraction Prop. of Eq.**

3. If \( k = m \) and \( k + w = 12 \), then \( m + w = 12 \). **Substitution Prop. of Eq.**

4. If \( B \) is a point in the interior of \( \angle AOB \), then \( m\angle AOB + m\angle BOC = m\angle AOC \). **Angle Add. Post.**

5. Fill in the missing information.

**Given:** \( AC = 36 \)

\[
\begin{align*}
x &= 3x - 2x + 1 \\
2x &= 2x + 1
\end{align*}
\]

a. \( AB + BC = AC \) \( \rightarrow \) **Substitution Prop. of Eq.**

b. \( 3x + 2x + 1 = 36 \) \( \rightarrow \) **Segment Add. Post.**

c. \( 5x + 1 = 36 \) \( \rightarrow \) **Substitution Prop. of Eq.**

d. \( 5x = 35 \) \( \rightarrow \) **Subtraction Prop. of Eq.**

e. \( x = 7 \) \( \rightarrow \) **Division Prop. of Eq.**

**Alternative Assessment**

Have students work in pairs. Each student should write a linear equation that can be solved for \( x \). Then have partners exchange equations and solve, justifying each step.
Multiple Choice

38. Which property justifies this statement?  
   D. Symmetric Property of Equality

   If \(4x = 16\), then \(16 = 4x\).
   A. Multiplication Property of Equality
   B. Transitive Property of Equality
   C. Reflexive Property of Equality

39. The Multiplication Property of Equality justifies which statement below?  
   H. If \(\frac{3}{4}x = 6\), then \(3x = 24\).
   F. If \(\frac{3}{4}x = 6\), then \(\frac{3}{4}x = 6\).
   G. If \(\frac{3}{4}x + 5 = 6\), then \(\frac{3}{4}x = 1\).
   J. If \(\frac{3}{4}x - 18 = 6\), then \(\frac{3}{4}x = 24\).

40. A transitive property justifies which statement below?  
   D. If \(a = b\) and \(b = c\), then \(a = c\).
   A. If \(a = 2\) and \(b = 3\), then \(a + b = 5\).
   B. If \(a = 2\) and \(b = 3\), then \(a < b\).
   C. If \(a = 2\) and \(b = 3\), then \(a = b\).

41. Which equation follows from \(\frac{1}{2}m + 1 = 10\) by the Multiplication Property of Equality?  
   F. \(m + 3 = 30\)
   G. \(\frac{1}{2}m = 9\)
   H. \(\frac{1}{2}m - 9 = 0\)
   J. \(m - 27 = 0\)

Short Response

42. In the diagram, \(x = 2y + 15\) and \(x + y = 120\).  
   a. Use a Property of Equality to explain why \(3y + 15 = 120\).
   b. Solve for \(y\). Justify each step. Then find the value of \(x\).

   Answers may vary. Sample: \(\angle AOD, \angle BOE\)

Mixed Review

Lesson 2-3

Reasoning Use logical reasoning to draw a conclusion.

43. If a student is having difficulty in class, then that student’s teacher is concerned. Elena is having difficulty in history class.  
   Elena’s teacher is concerned.

44. If a person has a job, then that person is earning money.  
   If a person is earning money, then that person can save money each week.  
   If a person has a job, then that person can save money each week.

Lesson 1-6

Use the diagram at the right and find each measure.

45. \(m\angle AOC = 80\)
46. \(m\angle AOD = 125\)
47. \(m\angle DOB = 65\)
48. \(m\angle BOE = 90\)

49. In the diagram, name an obtuse angle and a right angle.
   Answers may vary. Sample: \(\angle AOD, \angle BOE\)

Lesson 1-1

Find the next two terms in each sequence.

50. 19, 21.5, 24, 26.5
51. 3.4, 3.45, 3.456, 3.4567, 3.456789
52. -2.6, -18, 54, -162, 486
53. 8, -4, 2, -1, \(-\frac{1}{2}, -\frac{1}{4}\)