What You’ll Learn
• To prove and apply theorems about angles

... And Why
To find the measures of angles formed by the legs of a director’s chair, as in Exercise 11.

Fill in each blank.

4. Perpendicular lines are two lines that intersect to form □ right angles.
5. An angle is formed by two rays with the same endpoint.

The endpoint is called the □ of the angle. vertex

New Vocabulary
• theorem
• paragraph proof

Theorems About Angles

Hands-On Activity: Vertical Angles
• Draw two intersecting lines.
  Number the angles as shown.
• Fold ∠1 onto ∠2.
• Fold ∠3 onto ∠4.
• Make a conjecture about vertical angles.
  Vertical angles are congruent.

You can use deductive reasoning to show that a conjecture is true. The set of steps you take is called a proof. The statement that you prove true is a theorem. The Investigation above leads to a conjecture that becomes the following theorem.

Key Concepts
Theorem 2-1 Vertical Angles Theorem
Vertical angles are congruent.
∠1 ≅ ∠2 and ∠3 ≅ ∠4

In the proof of a theorem, a “Given” list shows you what you know from the hypothesis of the theorem. You prove the conclusion of the theorem. A diagram records the given information visually.

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Differentiated Instruction Solutions for All Learners

Special Needs
Have students perform the lesson Activity using lines that are not close to being perpendicular. Then have them cut out the angles and match them to reinforce the fact that vertical angles are congruent.

Below Level
Have students use protractors to verify the Vertical Angles Theorem. Actually measuring the angles also will reinforce the underlying algebraic argument in the paragraph proof on p. 111.

learning style: tactile
Here is what the start of many proofs will look like.

Given: __________

Prove: __________

Diagram that shows what you know

There are many forms of proofs. A **paragraph proof** is written as sentences in a paragraph. Here is a paragraph proof of Theorem 2-1.

**Proof:** By the Angle Addition Postulate, \( m\angle 1 + m\angle 3 = 180 \) and \( m\angle 2 + m\angle 3 = 180 \). By substitution, \( m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3 \). Subtract \( m\angle 3 \) from each side. You get \( m\angle 1 = m\angle 2 \), or \( \angle 1 \cong \angle 2 \).

You can use the Vertical Angles Theorem to solve for variables and find the measures of angles.

**Example 1** Using the Vertical Angles Theorem

**Gridded Response** Find the value of \( x \).

\[
4x = 3x + 35 \quad \text{Vertical angles are congruent.}
\]

Subtract \( 3x \) from each side.

\[
x = 35
\]

**Quick Check**

1. Find the measures of the labeled pair of vertical angles in the diagram above.
2. Find the measures of the other pair of vertical angles.
3. Check to see that adjacent angles are supplementary.

Find the value of \( x \).

**Additional Examples**

1. **Teaching Tip**

Ask: How is this solution method like writing a paragraph proof? How is it different? Sample: You work step-by-step, building on what you know; you can quickly see the list of steps.

**Key Concepts**

**Theorem 2-2** Congruent Supplements Theorem

If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

**Guided Instruction**

**Hands-On Activity**

Provide patty paper (which is also used to separate frozen hamburger patties) for students to fold for this activity.

**Technology Tip**

Students can explore the Vertical Angles Theorem with geometry software.

**Tactile Learners**

Have students copy the diagrams for the Vertical Angles and Congruent Supplements Theorems to reinforce each Given and the conclusions that follow.

**Alternative Method**

Work as a class to write the paragraph proof preceding Example 1 in two columns, justifying each step in the right column as in Lesson 2-4.

**Advanced Learners**

Have students prove the statement: “If two angles are congruent, then their supplements are congruent.”

This is the converse of the Congruent Supplements Theorem.

**English Language Learners**

Help students recognize that a **proof** is a set of steps to show a conjecture is true. Proofs use given information, definitions, postulates, and theorems as reasons that justify a step.
PROOF

Given:
\( \angle 1 \) and \( \angle 2 \) are supplementary.
\( \angle 3 \) and \( \angle 2 \) are supplementary.

Prove: \( \angle 1 \equiv \angle 3 \)

Proof: By the definition of supplementary angles, \( m\angle 1 + m\angle 2 = 180 \) and \( m\angle 3 + m\angle 2 = 180 \). By substitution, \( m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2 \). Subtract \( m\angle 2 \) from each side. You get \( m\angle 1 = m\angle 3 \), or \( \angle 1 \equiv \angle 3 \).

Quick Check

In the proof above, which Property of Equality allows you to subtract \( m\angle 2 \) from each side of the equation? Subtraction Property of Equality

Theorem 2-3 is like the Congruent Supplements Theorem. You can demonstrate its proof in Exercises 7 and 28.

Key Concepts

- **Theorem 2-3**
  - **Congruent Complements Theorem**
  - If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.
- **Theorem 2-4**
  - All right angles are congruent.
- **Theorem 2-5**
  - If two angles are congruent and supplementary, then each is a right angle.

You can complete proofs of Theorems 2-4 and 2-5 in Exercises 14 and 21, respectively.

EXERCISES

Practice and Problem Solving

### Practice by Example

Find the value of each variable.

1. \( 3x = 20 \)
2. \( (80 - x)^\circ = 75^\circ \)
3. \( x + 90^\circ = 4x^\circ \)

### GO for Help

Find the measures of the labeled angles in each exercise.

4. Exercise 1 \( 60, 60 \)
5. Exercise 2 \( 75, 105 \)
6. Exercise 3 \( 120, 120 \)
7. Developing Proof Complete this proof of one form of Theorem 2-3 by filling in the blanks.

If two angles are complements of the same angle, then the two angles are congruent.

- Given: $\angle 1$ and $\angle 2$ are complementary. $\angle 3$ and $\angle 2$ are complementary.

- Prove: $\angle 1 \cong \angle 3$

- Proof: By the definition of complementary angles, $m\angle 1 + m\angle 2 = \text{and} m\angle 3 + m\angle 2 = \text{.}$ Then $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$ by . Subtract $m\angle 2$ from each side. You get $m\angle 1 = \text{, or } \angle 1 \cong \angle 3.$

8. Writing How is a theorem different from a postulate? Sample: A theorem is proven and a postulate is assumed to be true.

9. Open-Ended Give an example of vertical angles in your home. Answers may vary. Sample: scissors

10. Reasoning Explain why this statement is true:

If $m\angle 1 + m\angle 2 = 180$ and $m\angle 3 + m\angle 2 = 180,$ then $\angle 1 \cong \angle 3.$ See margin.

11. Design The two back legs of the director’s chair pictured at the left meet in a $72^\circ$ angle. Find the measure of each angle formed by the two back legs. See margin

12. Algebra Find the value of each variable and the measure of each labeled angle.

- $x = 14, y = 15; 50, 50, 130$

13. 

14. Developing Proof Complete this proof of Theorem 2-4 by filling in the blanks.

All right angles are congruent.

- Given: $\angle X$ and $\angle Y$ are right angles.

- Prove: $\angle X \cong \angle Y$

- Proof: By the definition of $\angle X \cong \angle Y$, $m\angle X = 90$ and $m\angle Y = 90.$ By the Substitution Property, $m\angle X = \text{, or } \angle X \cong \angle Y.$ $m\angle Y$

15. Multiple Choice What is the measure of the angle formed by Park St. and Oak St.? C

A $35^\circ$ B $45^\circ$

C $55^\circ$ D $90^\circ$

16. Name two pairs of congruent angles in each figure. Justify your answers.

17. See margin.

19. Coordinate Geometry $\angle DOE$ contains points $D(2, 3), O(0, 0),$ and $E(5, 1).$ Find the coordinates of a point $F$ so that $\overline{OF}$ is a side of an angle that is adjacent and supplementary to $\angle DOE.$ Answers may vary. Sample: $(–5, –1)$
Exercise 8  When students are finished, ask: How is a theorem similar to a postulate? Sample: Both are true statements about geometric figures. Point out that students will use both postulates and theorems that are already proved to prove each new theorem.

Connection to Architecture
Exercise 9 Ask: What angles are used frequently in house and building design? Sample: right angle, straight angle Have students discuss where these angles appear.

Exercise 15 Have students identify what they are trying to find in the diagram. Then ask: What angle information are you given? one right angle and a 35° angle

Exercises 16–18 Have partners explain their reasoning to each other. Point out that being able to explain your reasoning to each other. Point out that being able to explain their reasoning to each other is like writing a good proof in geometry.

Exercises 27, 28 Do these exercises as a class activity. Students may refer back to the proofs in the lesson for ideas, if necessary.

Connection to Algebra
Exercise 30–32 Students must set up and solve a system of two equations.

10. If \( m\angle 1 + m\angle 2 = 180 \), and \( m\angle 2 + m\angle 3 = 180 \), then \( m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3 \) by subst. Subtr. \( m\angle 2 \) from each side \( m\angle 1 = m\angle 3 \) or \( \angle 1 \equiv \angle 3 \).

11. The two acute \( \angle \) have measure 72. The two obtuse \( \angle \) have measure 108.

16. \( \angle DOB \equiv \angle AOC \) and \( \angle DOA \equiv \angle BOC \) since they are vert. \( \angle \).

17. \( \angle EIG \equiv \angle FIH \) since all rt. \( \angle \) are \( \equiv \); \( \angle EIF \equiv \angle HIG \) since they are compl. of the same \( \angle \).

20. Coordinate Geometry \( \angle AOX \) contains points \( A(1, 3) \), \( O(0, 0) \), and \( X(4, 0) \).
   a. Find the coordinates of a point \( B \) so that \( \angle BOA \) and \( \angle AOX \) are adjacent complementary angles. \( \text{a-b. See margin.} \)
   b. Find the coordinates of a point \( C \) so that \( \angle OC \) is a side of a different angle that is adjacent and complementary to \( \angle AOX \).

21. Developing Proof Complete this proof of Theorem 2-5 by filling in the blanks.

If two angles are congruent and supplementary, then each is a right angle.

**Given:** \( \angle W \) and \( \angle V \) are congruent and supplementary.

**Prove:** \( \angle W \) and \( \angle V \) are right angles.

**Proof:**

\[ \angle W \] and \( \angle V \) are congruent, so \( m\angle W = m\angle A, \angle V \)

\( \angle W \) and \( \angle V \) are supplementary so \( m\angle W + m\angle V = \text{b, 180} \)

Substituting \( m\angle W \) for \( m\angle V \), you get \( m\angle W + m\angle W = 180 \), or \( 2m\angle W = 180 \).

By the Property of Equality, \( m\angle W = 90 \). Division

Since \( \angle W \equiv \angle V \), \( m\angle V = 90 \). Then both angles are \( \angle \) angles. right

22. Sports In the photograph, the wheels of the racing wheelchair are tilted so that \( \angle 1 \equiv \angle 2 \). What theorem can you use to justify the statement \( \angle 3 \equiv \angle 4 \)? Supplements of \( \angle \) are \( \equiv \).

23. \( \angle A \) is twice as large as its complement, \( \angle B \). \( m\angle A = 60 \), \( m\angle B = 30 \)

24. \( \angle A \) is half as large as its complement, \( \angle B \). \( m\angle A = 30 \), \( m\angle B = 60 \)

25. \( \angle A \) is twice as large as its supplement, \( \angle B \). \( m\angle A = 120 \), \( m\angle B = 60 \)

26. \( \angle A \) is half as large as twice its supplement, \( \angle B \). \( m\angle A = 90 \), \( m\angle B = 90 \)

**Proof**

27. Write a proof for this form of Theorem 2-2. See margin.

If two angles are supplements of congruent angles, then the two angles are congruent.

**Given:** \( \angle 1 \) and \( \angle 2 \) are supplementary.

\( \angle 3 \) and \( \angle 4 \) are supplementary.

\( \angle 2 \equiv \angle 4 \)

**Prove:** \( \angle 1 \equiv \angle 3 \)

28. Write a proof for this form of Theorem 2-3. See margin.

If two angles are complements of congruent angles, then the two angles are congruent.

**Given:** \( \angle 1 \) and \( \angle 2 \) are complementary.

\( \angle 3 \) and \( \angle 4 \) are complementary.

\( \angle 2 \equiv \angle 4 \)

**Prove:** \( \angle 1 \equiv \angle 3 \)

29. Paper Folding After you’ve done the Activity on page 110, answer these questions. a–b. It is the bisector of both angles.

a. How is the first fold line you make related to angles 3 and 4?

b. How is the second fold line you make related to angles 1 and 2?

c. How are the two fold lines related to each other? Give a convincing argument to support your answer. Sample: perpendicular; bisectors of two adjacent supplementary angles form two adjacent angles whose measures add to \( \frac{1}{2} \) \( \angle \), or 90.

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18. \( \angle KPJ \equiv \angle MPJ \) since they are marked \( \equiv \);

\( \angle KPL \equiv \angle MPL \) since they are suppl. of \( \equiv \) \( \angle \).

19. Answers may vary. Sample: \((-5, -1)\)

20. a. Answers may vary. B can be any point on the positive y-axis. Sample: \((0, 5)\).

b. Answers may vary. Sample: \((3, -1)\)
**Algebra** Find the value of each variable and the measure of each labeled angle.

30. \(x = 30, y = 90; 60, 120, 60\)
31. \(x = 35, y = 70; 70, 110, 70\)
32. \(x = 50, y = 20; 80, 100, 80\)

**Test Prep**

**Gridded Response**

Find the measure of each angle.

33. an angle with measure 8 less than the measure of its complement \(41^\circ\)
34. one angle of a pair of complementary vertical angles \(45^\circ\)
35. an angle with measure three times the measure of its supplement \(135^\circ\)

Use the diagram at the right to find the measure of each of the following angles.

36. \(\angle 1 \ 20^\circ\)
37. \(\angle 2 \ 90^\circ\)
38. \(\angle 3 \ 70^\circ\)
39. \(\angle 4 \ 110^\circ\)

**Alternative Assessment**

Write this statement on the board. 
*If two angles are congruent and complementary, then each has a measure of 45°.*

Have partners draw and label a diagram that fits the theorem and write a paragraph proof of the theorem.

Alternatively, if students are not yet ready to write a paragraph proof, ask them to write a full explanation of why the theorem makes sense. Writing an explanation is similar to writing a proof, but the language may be less intimidating to students.

**Test Prep**

A sheet of blank grids is available in the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.