**Exploring Spherical Geometry**

Students may think that Euclidean geometry is the only geometry possible to study. This Extension shows how the basic ideas of point, line, and plane are defined on a sphere. Euclidean geometry is actually the first of many geometries students may encounter.

**Guided Instruction**

**Tactile Learners**

Have students slice an orange to model the intersection of a sphere and a plane.

**Math Tip**

The order of postulates and theorems in this textbook is different from the order in which Euclid presented them.

**Connection to History**

Sailors, explorers, and mapmakers rely on accurate measurements of longitude and latitude. Have students research the history of longitude measurement and how accurate methods were developed.

Euclidean geometry is the basis for high school geometry courses. Euclidean geometry is the geometry of flat planes, straight lines, and points. In spherical geometry a “plane” is the curved surface of a sphere and a “line” is a great circle. (A great circle is the intersection of a sphere and a plane that contains the center of the sphere.)

1. **ACTIVITY**

Lines of latitude and longitude are used to identify positions on Earth. Find these lines on a globe. Which of these lines are great circles?

All lines of longitude are great circles. The equator is the only line of latitude that is a great circle. All other lines of latitude are circles smaller than a great circle.

In Euclidean geometry,

*Through a point not on a line, there is one and only one line parallel to the given line.*

This statement is sometimes called Euclid’s Parallel Postulate. Since only great circles are lines in spherical geometry, two lines always intersect. In spherical geometry, the Parallel Postulate is quite different:

*Through a point not on a line, there is no line parallel to the given line.*

2. **ACTIVITY**

The diagram at the right shows that any two lines on a sphere intersect at two points. Locate the points of intersection of lines of longitude on Earth. What is special about these points?

Lines of longitude intersect at the North and South Poles. The poles are on a line that passes through the center of Earth. Thus, the poles lie on Earth’s axis and are the endpoints of a diameter of Earth.

One result of Euclid’s Parallel Postulate is the Triangle Angle-Sum Theorem of Lesson 3-3. Something quite different happens in spherical geometry as a result of the spherical-geometry Parallel Postulate.
Hold a string taut between any two points on a sphere. The string forms an arc that is part of a great circle. Make three such arcs to form a triangle on the sphere.

Three such triangles are shown below. Find the sum of the measures of the angles of each of the triangles.

In the first triangle, the sum of the angle measures is 190. In the second triangle, it is 210, and in the third triangle the sum is 280.

EXERCISES

Draw a sketch to illustrate each property of spherical geometry. How does each property compare to what is true in Euclidean geometry?

1. There are pairs of points on a sphere through which more than one line can be drawn. 1–3. See right.
2. A triangle can have more than one right angle.
3. You can draw two equiangular triangles such that they have different angle measures.

In Exercises 4 and 5, draw a counterexample to show that each of these properties of Euclidean geometry is not true in spherical geometry. 4–7. See margin.

4. Two lines that are perpendicular to the same line do not intersect.
5. If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.
6. The figure at the right appears to show parallel lines on a sphere. Explain why this is not so.
7. Explain why a piece of the top circle in the figure is not a line segment. (Hint: What must be true of line segments in spherical geometry?)

Each of the following statements is true in Euclidean geometry. Does it seem to be true in spherical geometry? Make figures on a globe, ball, or balloon to support your answer. 8–9. See margin.

8. Vertical angles are congruent.
9. Through a point on a line $\ell$ there exists one and only one line perpendicular to $\ell$.

4–5. Answers may vary. Samples are given: 4.

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