Midsegments of Triangles

Lesson Preview

What You’ll Learn

• To use properties of midsegments to solve problems

... And Why

To use indirect measurement to find the length of a lake, as in Example 3

Check Skills You’ll Need

Find the coordinates of the midpoint of each segment.

1. \( \overline{AB} \) with \( A(-2,3) \) and \( B(4,1) \) \( (1,2) \)
2. \( \overline{CD} \) with \( C(0,5) \) and \( D(3,6) \) \( \left( \frac{3}{2}, \frac{11}{2} \right) \)
3. \( \overline{EF} \) with \( E(-4,6) \) and \( F(3,10) \) \(-\frac{5}{2}, \frac{11}{2}\)
4. \( \overline{GH} \) with \( G(7,10) \) and \( H(-5,-8) \) \( (1,1) \)

Find the slope of the line containing each pair of points.

5. \( A(-2,3) \) and \( B(3,1) \) \( \frac{-5}{5} \)
6. \( C(0,5) \) and \( D(3,6) \) \( \frac{5}{3} \)
7. \( E(-4,6) \) and \( F(3,10) \) \( \frac{11}{7} \)
8. \( G(7,10) \) and \( H(-5,-8) \) \( \frac{19}{12} \)

New Vocabulary

• midsegment
• coordinate proof

Using Properties of Midsegments

Hands-On Activity: Midsegments of Triangles

Draw, label, and cut out a large scalene triangle. Do the same with other right, acute, and obtuse triangles. Label the vertices \( A, B, \) and \( C. \)

• For each triangle fold \( A \) onto \( C \) to find the midpoint of \( \overline{AC}. \) Do the same for \( \overline{BC}. \) Label the midpoints \( L \) and \( N, \) then draw \( \overline{LN}. \)

• Fold each triangle on \( \overline{LN}. \)

1. \( LN = \frac{1}{2} AB; \) Explanations may vary.
2. \( A \) to \( C. \) Fold \( B \) to \( C. \)
3. How does \( LN \) compare to \( AB? \)
4. Make a conjecture about how the segment joining the midpoints of two sides of a triangle is related to the third side of the triangle.

See left.

In \( \triangle ABC \) above, \( \overline{LN} \) is a triangle midsegment. A midsegment of a triangle is a segment connecting the midpoints of two sides.

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Objectives

1. To use properties of midsegments to solve problems

Examples

1. Finding Lengths
2. Identifying Parallel Segments
3. Real-World Connection

Math Background

Euclid did not use coordinate geometry to prove any theorems. The Triangle Midsegment Theorem can be proved without coordinate geometry, but the proof requires theorems concerning parallelograms that are not presented in this text until Chapter 6.

More Math Background: p. 256C

Lesson Planning and Resources

See p. 256E for a list of the resources that support this lesson.

Bell Ringer Practice

Check Skills You’ll Need

For intervention, direct students to:

Finding the Midpoint of a Segment

Lesson 1-8: Example 3
Extra Skills, Word Problems, Proof Practice, Ch. 1

Slope

Algebra Review, p. 165

Differentiated Instruction Solutions for All Learners

Special Needs

In the Hands-On Activity, some students may not see why folding \( A \) onto \( C \) marks the midpoint of \( \overline{AC} \) because of its orientation. Demonstrate by folding the corners of a rectangular piece of paper.

Below Level

To eliminate the fractions in proving Theorem 5-1, let the respective coordinates of points \( Q \) and \( P \) be \((2a, 0)\) and \((2b, 2c)\).

learning style: visual learning style: verbal
One way to prove the Triangle Midsegment Theorem is to use coordinate geometry and algebra. This style of proof is called a coordinate proof. You begin the proof by placing a triangle in a convenient spot on the coordinate plane. You then choose variables for the coordinates of the vertices.

**Proof of Theorem 5-1**

**Given:** \( R \) is the midpoint of \( \overline{OP} \). 
\( S \) is the midpoint of \( \overline{OQ} \).

**Prove:** \( RS \parallel \overline{OQ} \) and \( RS = \frac{1}{2}OQ \)

- Use the Midpoint Formula to find the coordinates of \( R \) and \( S \).
  \[
  R: \left( \frac{0 + b}{2}, \frac{0 + c}{2} \right) = \left( \frac{b}{2}, \frac{c}{2} \right) 
  
  S: \left( \frac{a + b}{2}, \frac{0 + c}{2} \right) = \left( \frac{a + b}{2}, \frac{c}{2} \right) 
  
- To prove that \( RS \parallel \overline{OQ} \) are parallel, show that their slopes are equal. Because the y-coordinates of \( R \) and \( S \) are the same, the slope of \( RS \) is zero. The same is true for \( \overline{OQ} \). Therefore, \( RS \parallel \overline{OQ} \).

- Use the Distance Formula to find \( RS \) and \( OQ \).
  \[
  RS = \sqrt{\left( \frac{a + b}{2} - \frac{b}{2} \right)^2 + \left( \frac{c}{2} - \frac{c}{2} \right)^2} 
  = \sqrt{\left( \frac{a - b}{2} \right)^2 + 0^2} = \sqrt{\left( \frac{a}{2} \right)^2} = \frac{a}{2} 
  
  OQ = \sqrt{(a - 0)^2 + (0 - 0)^2} 
  = \sqrt{a^2 + 0^2} = a 
  
Therefore, \( RS = \frac{1}{2}OQ \).

**EXAMPLE 1 Finding Lengths**

In \( \triangle EFG \), \( H \), \( J \), and \( K \) are midpoints.
Find \( HJ \), \( JK \), and \( FG \).

- \( HJ = \frac{1}{2}EG \) or \( \frac{1}{2}(100); HJ = 50 \)
- \( JK = \frac{1}{2}EF \) or \( \frac{1}{2}(60); JK = 30 \)
- \( HK \) or \( 40 = \frac{1}{2}FG; FG = 80 \)

**Quick Check**

\( AB = 10 \) and \( CD = 18 \). Find \( EB \), \( BC \), and \( AC \).

- \( EB = 9; BC = 10; AC = 20 \)

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**Vocabulary Tip**

- **The Midpoint Formula**
  \[
  \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) 
  
- **The Distance Formula**
  \[
  \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} 
  
**Math Tip**

Discuss as a class why the vertices in the proof of the Triangle Midsegment Theorem are labeled \( O(0, 0) \), \( Q(a, 0) \), and \( P(b, c) \). Explain that translating, rotating, or reflecting a triangle so that two of its vertices are at \( (0, 0) \) and \( (a, 0) \) simplifies using the Midpoint and Distance Formulas.

**Hands-On Activity**

Have students place labels for the vertices inside the triangle on both sides of the paper so they will appear on the cut-out figures. Instruct students to label the obtuse or right angle vertex \( C \) to ensure that the first folded triangle lies inside \( \triangle ABC \).

**Connection to Algebra**

The proof of the Triangle Midsegment Theorem uses the Midpoint and Distance Formulas from Chapter 1 and the calculation of slope from Chapter 3. Ask:

Why are variables used in the proof instead of numbers? Using numbers proves the theorem for one set of points. Because any number can be substituted for a variable, using variables proves the theorem for all sets of points.

**Math Tip**

Discuss as a class why the vertices in the proof of the Triangle Midsegment Theorem are labeled \( O(0, 0) \), \( Q(a, 0) \), and \( P(b, c) \). Explain that translating, rotating, or reflecting a triangle so that two of its vertices are at \( (0, 0) \) and \( (a, 0) \) simplifies using the Midpoint and Distance Formulas.

**Auditory Learners**

Have students read through Example 1 in small groups. Then ask volunteers to explain how the example applies the Triangle Midsegment Theorem.
**Identifying Parallel Segments**

In \(\triangle DEF\), \(A\), \(B\), and \(C\) are midpoints. Name pairs of parallel segments.

The midsegments are \(AB\), \(BC\), and \(CA\).

By the Triangle Midsegment Theorem, \(AB \parallel DP\), \(BC \parallel EF\), and \(AC \parallel LE\).

**Critical Thinking** Find \(m\angle VUZ\). Justify your answer.

65; \(UV \parallel XY\) so \(\angle VUZ\) and \(\angle YXZ\) are corr. and \(\cong\).

You can use the Triangle Midsegment Theorem to find lengths of segments that might be difficult to measure directly.

**Real-World Connection**

Dean plans to swim the length of the lake, as shown in the photo. How far would Dean swim?

Here is what Dean does to find the distance he would swim across the lake.

Step 1: He measures his stride and adjusts it so that it averages about 3 ft.

Step 2: Then he begins at the left edge of the lake (first diagram). He paces 35 strides along the edge of the lake and sets a stake.

Step 3: He paces 35 more strides in the same direction and sets another stake.

Step 4: He paces to where his swim will end at the other side of the lake, counting 236 strides.

Step 5: Then (second diagram) he paces 118 strides, or half the distance, back towards the second stake.

Step 6: He paces to the first stake, counting 128 strides.

Step 7: He converts strides to feet.

\[128 \text{ strides} \times \frac{3 \text{ ft}}{1 \text{ stride}} = 384 \text{ ft}\]

Step 8: He uses Theorem 5-1. The distance across the lake is twice the length of the midsegment.

\[2(384 \text{ ft}) = 768 \text{ ft}\]

Dean would swim approximately 768 ft.

**Quick Check**

3. \(CD\) is a new bridge being built over a lake as shown. Find the length of the bridge. \(1320 \text{ ft}\)

b. How long is the bridge in miles? \(\frac{3}{4} \text{ mi}\)

**Additional Examples**

1. In \(\triangle XYZ\), \(M\), \(N\), and \(P\) are midpoints. The perimeter of \(\triangle MNP\) is 60. Find \(NP\) and \(YZ\).

2. Find \(m\angle AMN\) and \(m\angle ANM\).

3. Explain why Dean could use the Triangle Midsegment Theorem to measure the length of the lake. He paced between the midpoints of two sides of a triangle.

**Resources**
- Daily Notetaking Guide 5-1
- Daily Notetaking Guide 5-1—Adapted Instruction

**Closure**

The perimeter of a triangle is 78 ft. Find the perimeter of the triangle formed by its midsegments. \(39 \text{ ft}\)
**Chapter 5**

### Practice and Problem Solving

**EXERCISES**

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

<table>
<thead>
<tr>
<th>Practice by Example</th>
<th>Mental Math</th>
<th>Find the value of $x$.</th>
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<tbody>
<tr>
<td>Example 1 (page 260)</td>
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<td>1. $9$</td>
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<td>5. $232$</td>
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<td></td>
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</tbody>
</table>

Points $E$, $D$, and $H$ are midpoints of $\triangle TUV$. $UV = 80$, $TV = 100$, and $HD = 80$.


**Example 2** (page 261)

Identify pairs of parallel segments in each diagram.

11. $UW \parallel TX; UV \parallel VX$; $YW \parallel TV$

12. $GJ \parallel FK; JL \parallel HF; GL \parallel HK$

13. a. In the figure at the right, identify pairs of parallel segments. $ST \parallel PR; SU \parallel QR; UT \parallel PQ$
   
   b. If $m \angle QST = 40$, find $m \angle QPR$.
   
   $m \angle QPR = 40$

**Example 3** (page 261)

Name the segment that is parallel to the given segment.

14. $\overline{AB} \parallel \overline{EF}$
   15. $\overline{BC} \parallel \overline{FG}$

16. $\overline{EF} \parallel \overline{AB}$
   17. $\overline{CA} \parallel \overline{EG}$

18. $\overline{GE} \parallel \overline{AC}$
   19. $\overline{FG} \parallel \overline{CB}$

20. **Indirect Measurement** Kate wants to paddle her canoe across the lake. To determine how far she must paddle, she paced out a triangle, counting the number of strides, as shown.

   a. If Kate’s strides average 3.5 ft, what is the length of the longest side of the triangle?
   
   b. What distance must Kate paddle across the lake? 437.5 ft

20a. 1050 ft

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*For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*
21. **Architecture** The triangular face of the Rock and Roll Hall of Fame in Cleveland, Ohio, is isosceles. The length of the base is 229 ft 6 in. What is the length of the highlighted segment? **114 ft 9 in.**

**b. Writing** Explain your reasoning. See left.

**21b. Answers may vary. Sample:** The highlighted segment is a midsegment of the triangular face of the building.

**X** is the midpoint of **UV**. **Y** is the midpoint of **UW**.

22. If $\angle UXY = 60$, find $m\angle U$. **60**

23. If $\angle UVY = 45$, find $m\angle UYX$. **45**

24. If $XY = 50$, find $VW$. **100**

25. If $VW = 110$, find $XY$. **55**

**26. Coordinate Geometry** The coordinates of the vertices of a triangle are $E(1, 2)$, $F(5, 6)$, and $G(3, -2)$. 

a. $H(2, 0)$; $J(4, 2)$ b-c. See margin.

b. Find the coordinates of $H$, the midpoint of $EG$, and $I$, the midpoint of $FG$.

c. Verify that $HI \parallel EF$.

27. $\triangle IJH = 18\frac{1}{2}$

28. $\triangle FGH = 37$

29. **Multiple Choice** Marita is designing a kite to look like the one on the left. Its diagonals are to measure 64 cm and 90 cm. She will use ribbon to connect the midpoints of its sides. How much ribbon will Marita need? C $A$ 77 cm $B$ 122 cm $C$ 154 cm $D$ 308 cm

**30.** $GP$ $x = 30$ $x = 60$

**31.** $GP$ $x = 50$

**32.** $x = 10$

**33.** $x = 2x + 1$

$x = 6; y = 6\frac{1}{2}$

**34.** If $DF = 24$, $BC = 6$, and $DB = 8$, find the perimeter of $\triangle ADF$. **52**

**35. Algebra** If $BE = 2x + 6$ and $DF = 5x + 9$, find the value of $x$, then find $DF$. $x = 3; DF = 24$

**36. Algebra** If $EC = 3x - 1$ and $AD = 5x + 7$, find the value of $x$, then find $EC$. $x = 9; EC = 26$

26. **b.** Slope of $HJ = \frac{2}{2} = 1$; slope of $EF = \frac{4}{4} = 1$; therefore $HJ \parallel EF$.

c. $HJ = \sqrt{2^2 + 2^2} = 2\sqrt{2}$; $EF = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2};$ therefore $HJ = \frac{1}{2} EF$. 

4. **Assess & Reteach**

**Lesson Quiz**

In $\triangle GHI$, $R$, $S$, and $T$ are midpoints.

**Problem Solving Hint**

The highlighted segment is halfway up the face of the Rock and Roll Hall of Fame.

1. Name all the pairs of parallel sides of $\triangle GHI$ and $\triangle RST$.

2. If $GH = 20$ and $HI = 18$, find $RT$. **9**

3. If $RH = 7$ and $RS = 5$, find $ST$. **7**

4. If $m\angle G = 60$ and $m\angle I = 70$, find $m\angle GTR$. **70**

5. If $m\angle H = 50$ and $m\angle I = 66$, find $m\angle ITS$. **64**

6. If $m\angle G = m\angle H = m\angle I$ and $RT = 15$, find the perimeter of $\triangle GHI$. **90**

**Alternative Assessment**

Draw the figure below on the board. Label the vertices of the large triangle and the midpoints of the sides. Name the triangles and the midsegments.

Have students use the given information to find the lengths of the sides of $\triangle DEF$ and the measures of the angles of $\triangle ABC$. Then have students explain in writing how they found the measures of the sides and angles.
50. Answers may vary.
Sample: Draw $CA$ and extend $CA$ to $P$ so that $CA = AP$. Find $B$, the midpt. of $PD$. Then, by the Triangle Midsegment Thm., $\overline{AB} \parallel \overline{CD}$ and $AB = \frac{1}{2} CD$.

39. $\triangle UTS$; Proofs may vary.
Sample: $VS \equiv SY$, $YT \equiv TZ$, and $VU \equiv UZ$ because $S$, $T$, and $U$ are midpts. of the respective sides; $ST = \frac{1}{2} VZ$ so $\overline{ST} \equiv \overline{VU} \equiv \overline{UZ}$; $SU = \frac{1}{2} YZ$ so $\overline{SU} \equiv \overline{YZ}$; and $TU = \frac{1}{2} VY$ so $\overline{TU} \equiv \overline{SY} \equiv \overline{SV}$; therefore $\triangle YST \equiv \triangle TUZ \equiv \triangle SVU \equiv \triangle UTS$ by SSS.

Mixed Review

Lesson 4-7
Name a pair of overlapping congruent triangles in each diagram. State whether the triangles are congruent by SSS, SAS, ASA, AAS, or HL.

47. $\triangle SXT \equiv \triangle TYS$; SAS
48. $\triangle ADC \equiv \triangle EBC$; ASA
49. $\triangle KLQ \equiv \triangle PNR$; HL

Lesson 3-6
Graph each line.

50. $y = x + 2$
51. $y = 3x - 2$
52. $y = -x - 5$

Lesson 3-2
Determine the value of $x$ for which $\ell \parallel m$.

53. $x = 4$ or $x = 2$
54. $x = 5$ or $x = 2$
55. $x = 3$ or $x = 3$

Test Prep

Q and P are midpoints of two sides of $\triangle RST$.

40. What is $RS$? 248
41. What is $TQ$? 174
42. What is $TS$? 418
43. What is $m\angle ABC$? 70
44. What is $m\angle D$? 40
45. What is $m\angle A$? 70
46. What is $m\angle CBE$? 40

Challenge

37. Open-Ended Explain how you could use the Triangle Midsegment Theorem as the basis for this construction. Draw $\overline{CD}$. Draw point $A$ not on $\overline{CD}$. Construct $\overline{AB}$ so that $\overline{AB} \parallel \overline{CD}$ and $AB = \frac{1}{2} CD$. See margin.

38. Coordinate Geometry In $\triangle GHI$, $K(2, 3)$ is the midpoint of $\overline{GH}$, $L(4, 1)$ is the midpoint of $\overline{HI}$, and $M(6, 2)$ is the midpoint of $\overline{GI}$. Find the coordinates of $G$, $H$, and $J$. $G(4, 4)$; $H(0, 2)$; $J(8, 0)$

Proof 39. Complete the prove statement and then write a proof.

Given: $S$, $T$, and $U$ are midpoints.

Prove: $\triangle YST \equiv \triangle TUZ \equiv \triangle SVU \equiv \Box$.

See margin.