5-3

1. Plan

Objectives

1 To identify properties of perpendicular bisectors and angle bisectors

2 To identify properties of medians and altitudes of a triangle

Examples

1 Finding the Circumcenter

2 Real-World Connection

3 Finding Lengths of Medians

4 Identifying Medians and Altitudes

Professional Development

Math Background

The theorems in this lesson can be related to Ceva's Theorem, which Giovanni Ceva published in 1678: Let sides $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ of $\triangle ABC$ be divided at X, Y, and Z respectively. Then $\overline{AZ}$, $\overline{BY}$, and $\overline{CX}$ are concurrent if and only if $\frac{AX}{XB} \cdot \frac{BZ}{ZC} \cdot \frac{CY}{YA} = 1$.
Concurrency theorems will be applied later to inscribed and circumscribed circles and to the study of centroids in physics.

More Math Background: p. 256C

Lesson Planning and Resources

See p. 256E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Constructing Perpendicular Bisectors

Lesson 1-7: Example 3

Extra Skills, Word Problems, Proof Practice, Ch. 1

Constructing Angle Bisectors

Lesson 1-7: Example 5

Extra Skills, Word Problems, Proof Practice, Ch. 1

5-3 Concurrent Lines, Medians, and Altitudes

What You'll Learn

- To identify properties of perpendicular bisectors and angle bisectors
- To identify properties of medians and altitudes of a triangle

. . . And Why

To find a location in a backyard for the largest possible swimming pool, as in Example 2

Check Skills You'll Need

GO for Help Lesson 1-7

For Exercises 1–2, draw a large triangle. Construct each figure. 1–4. See back of book.

1. an angle bisector

2. a perpendicular bisector of a side

3. Draw $\overline{GH}$. Construct $\overleftrightarrow{CD} \perp \overline{GH}$ at the midpoint of $\overline{GH}$.

4. Draw $\overleftrightarrow{AB}$ with a point E not on $\overleftrightarrow{AB}$. Construct $\overleftrightarrow{EF} \perp \overleftrightarrow{AB}$.

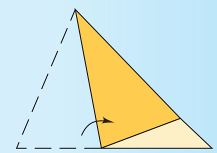
New Vocabulary

- concurrent
- point of concurrency
- circumcenter of a triangle
- circumscribed about
- incenter of a triangle
- inscribed in
- median of a triangle
- centroid
- altitude of a triangle
- orthocenter of a triangle

1 Properties of Bisectors

Hands-On Activity: Paper Folding Bisectors

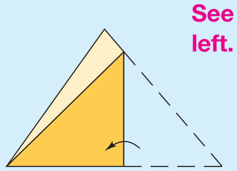
- Draw and cut out five different triangles: two acute, two right, and one obtuse.
- Step 1: Use paper folding to create the angle bisectors of each angle of an acute triangle. What do you notice about the angle bisectors?
- Step 2: Repeat Step 1 with a right triangle and an obtuse triangle. Does your discovery from Step 1 still hold true?



Folding an Angle Bisector

1. Make a conjecture about the bisectors of the angles of a triangle. See left.

1. The bisectors of the ∠s of a △ meet at a point inside the △.



Folding a Perpendicular Bisector

- Step 3: Use paper folding to create the perpendicular bisector of each side of an acute triangle. What do you notice about the perpendicular bisectors?
- Step 4: Repeat Step 3 with a right triangle. What do you notice?

2. Make a conjecture about the perpendicular bisectors of the sides of a triangle. See left.

2. The ⊥ bis. of the sides of a △ intersect at a point that might fall inside, outside, or on the △.

Differentiated Instruction Solutions for All Learners

Special Needs L1

For Example 2, have students copy the diagram. Using a compass, have them choose other centers and draw circles as large as possible that lie within the triangle. They will not find a larger circle.

learning style: tactile

Below Level L2

Have students use a compass or algebra to confirm that point (2, 3) is the center of the circle that contains points O, P, and S in Example 1.

learning style: tactile

When three or more lines intersect in one point, they are **concurrent.** The point at which they intersect is the **point of concurrency.** For any triangle, four different sets of lines are concurrent. Theorems 5-6 and 5-7 tell you about two of them.

Key Concepts

Theorem 5-6

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

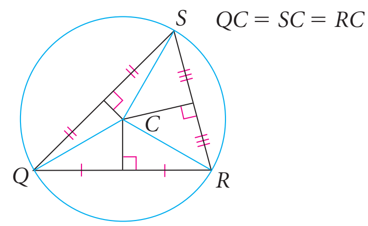
Theorem 5-7

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides.

You will prove these theorems in the exercises.

This figure shows $\triangle QRS$ with the perpendicular bisectors of its sides concurrent at C. The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter of the triangle.**

Points Q, R, and S are equidistant from C, the circumcenter. The circle is **circumscribed about** the triangle.



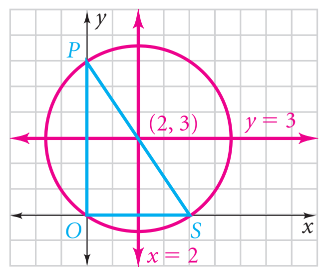
Vocabulary Tip

The prefix circum is Latin for "around" or "about."

1 EXAMPLE Finding the Circumcenter

Coordinate Geometry Find the center of the circle that you can circumscribe about $\triangle OPS$.

Two perpendicular bisectors of sides of $\triangle OPS$ are $x = 2$ and $y = 3$. These lines intersect at $(2, 3)$. This point is the center of the circle.

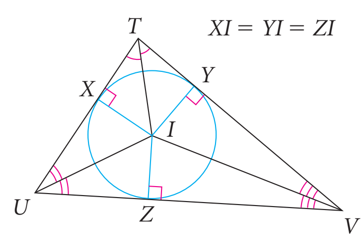


Quick Check 1 **a.** Find the center of the circle that you can circumscribe about the triangle with vertices $(0, 0)$, $(-8, 0)$, and $(0, 6)$. **(−4, 3)**

b. Critical Thinking In Example 1, explain why it is not necessary to find the third perpendicular bisector.
Thm. 5-6: All of the ⊥ bis. of the sides of a △ are concurrent.

This figure shows $\triangle UTV$ with the bisectors of its angles concurrent at I. The point of concurrency of the angle bisectors of a triangle is called the **incenter of the triangle.**

Points X, Y, and Z are equidistant from I, the incenter. The circle is **inscribed in** the triangle.



2. Teach

Guided Instruction

Hands-On Activity

Students may construct the angle bisectors and perpendicular bisectors using techniques they learned in Lesson 1-7.

Teaching Tip

When discussing Theorems 5-6 and 5-7, emphasize that the point of concurrency is equidistant from *vertices* for *perpendicular bisectors* and equidistant from *sides* for *angle bisectors.*

1 EXAMPLE Connection to Algebra

Remind students that the equation of a horizontal line is $y = a$ and the equation of a vertical line is $x = a$.

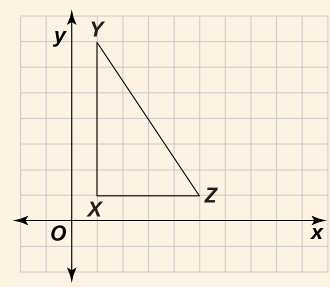
2 EXAMPLE Diversity

Remember that some students have little or no experience with houses that have yards big enough to hold a swimming pool.

PowerPoint

Additional Examples

1 Find the center of the circle that circumscribes $\triangle XYZ$. **(3, 4)**



Advanced Learners L4

Have students investigate Ceva's Theorem and how it can be used to prove Theorem 5-8.

learning style: verbal

English Language Learners ELL

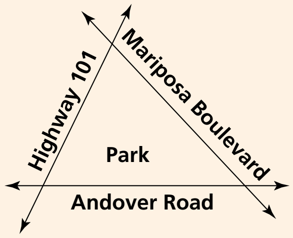
The terms *circumscribe* and *inscribe* can be related to their prefixes: *circum-* meaning around and *in-* meaning within. Students also need to understand the difference between *collinear* and *concurrent.*

learning style: verbal

PowerPoint

Additional Examples

2 City planners want to locate a fountain equidistant from three straight roads that enclose a park. Explain how they can find the location.



Locate the fountain at the point of concurrency of the angle bisectors of the triangle formed by the three roads.

Guided Instruction

Tactile Learners

Students can use paper-folding techniques to find altitudes and medians of triangles here and in Exercise 25.

Connection to Physical Science

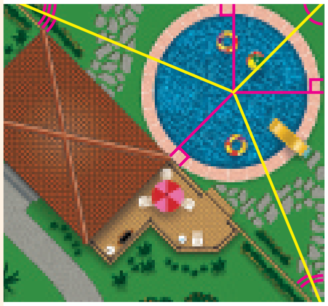
Have students read the Dorling Kindersly (DK) Activity Lab on pages 302–303, and do the Activity involving the centroid as a point of balance.

Teaching Tip

The proofs of Theorems 5-8 and 5-9 are postponed until students have the tools necessary to complete them.

3 EXAMPLE Math Tip

Point out that another way to state Theorem 5-8 is that each median is broken into segments that have a ratio of 2 : 1. This can help students use mental math to find lengths. Ask: *If* BD $= 22$, *what does* DE *equal?* **11**



2 EXAMPLE **Real-World Connection**

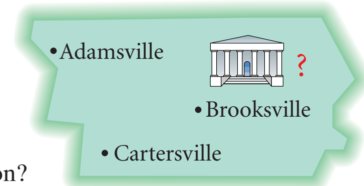
Pools The Jacksons want to install the largest possible circular pool in their triangular backyard. Where would the largest possible pool be located?

Locate the center of the pool at the point of concurrency of the angle bisectors. This point is equidistant from the sides of the yard. If you choose any other point as the center of the pool, it will be closer to at least one of the sides of the yard, and the pool will be smaller.

Quick Check

2a. Draw segments connecting the towns. Build the library at the inters. pt. of the ⊥ bisectors of the segments.

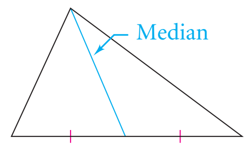
2 **a.** The towns of Adamsville, Brooksville, and Cartersville want to build a library that is equidistant from the three towns. Trace the diagram and show where they should build the library. **See left.**



b. What theorem did you use to find the location?
The ⊥ bisectors of the sides of a △ are concurrent at a point equidistant from the vertices.

2 Medians and Altitudes

A **median of a triangle** is a segment whose endpoints are a vertex and the midpoint of the opposite side.

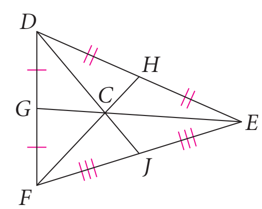


Key Concepts

Theorem 5-8

The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side.

$DC = \frac{2}{3}DJ \qquad EC = \frac{2}{3}EG \qquad FC = \frac{2}{3}FH$

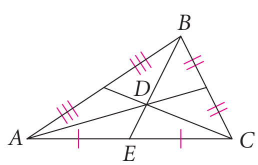


Test-Taking Tip

If you don't remember the meaning of a term, like centroid, the diagram may give a clue.

In a triangle, the point of concurrency of the medians is the **centroid.** The point is also called the center of gravity of a triangle because it is the point where a triangular shape will balance. (See DK Activity Lab, page 303.) You will prove Theorem 5-8 in Chapter 6.

3 EXAMPLE **Finding Lengths of Medians**



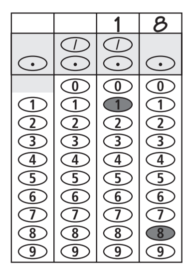
Gridded Response In $\triangle ABC$ at the left, D is the centroid and $DE = 6$. Find BE.

Since D is a centroid, $BD = \frac{2}{3}BE$ and $DE = \frac{1}{3}BE$.

$\frac{1}{3}BE = DE$

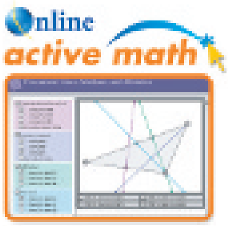
$\frac{1}{3}BE = 6$ **Substitute 6 for *DE*.**

$BE = 18$



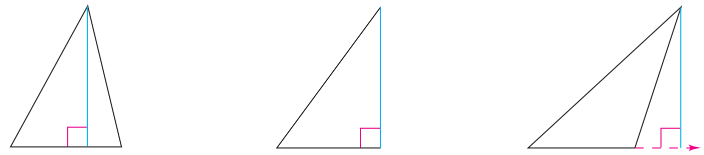
Quick Check

3 Find BD. Check that $BD + DE = BE$. **12**



For: Concurrent Lines Activity
Use: Interactive Textbook, 5-3

An **altitude of a triangle** is the perpendicular segment from a vertex to the line containing the opposite side. Unlike angle bisectors and medians, an altitude of a triangle can be a side of a triangle or it may lie outside the triangle.

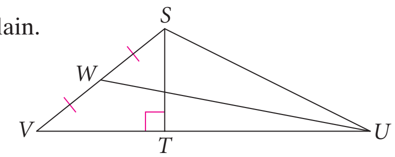


Acute Triangle: Altitude is inside. | Right Triangle: Altitude is a side. | Obtuse Triangle: Altitude is outside.

4 EXAMPLE Identifying Medians and Altitudes

Is $\overline{ST}$ a median, an altitude, or neither? Explain.

$\overline{ST}$ is a segment extending from vertex S to the side opposite S. Also, $\overline{ST} \perp \overline{VU}$. $\overline{ST}$ is an altitude of $\triangle VSU$.



Quick Check 4 Is $\overline{UW}$ a median, an altitude, or neither? Explain.
Median; $\overline{UW}$ is a segment drawn from vertex U to the midpt. of the opp. side.

The lines containing the altitudes of a triangle are concurrent at the **orthocenter of the triangle.** A proof of this theorem appears in Chapter 6.

Key Concepts

Theorem 5-9

The lines that contain the altitudes of a triangle are concurrent.

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

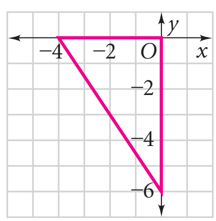
A Practice by Example

Example 1 (page 273)

GO for Help

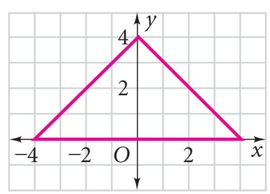
Coordinate Geometry **Find the center of the circle that you can circumscribe about each triangle.**

1.



(−2, −3)

2.



(0, 0)

Coordinate Geometry **Find the center of the circle that you can circumscribe about $\triangle ABC$.**

3. $A(0, 0)$ $B(3, 0)$ $C(3, 2)$ **$\left(1\frac{1}{2}, 1\right)$**

4. $A(0, 0)$ $B(4, 0)$ $C(4, -3)$ **$\left(2, -1\frac{1}{2}\right)$**

5. $A(-4, 5)$ $B(-2, 5)$ $C(-2, -2)$ **$\left(-3, 1\frac{1}{2}\right)$**

6. $A(-1, -2)$ $B(-5, -2)$ $C(-1, -7)$ **$\left(-3, -4\frac{1}{2}\right)$**

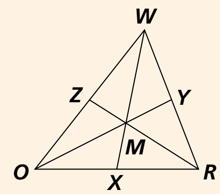
7. $A(1, 4)$ $B(1, 2)$ $C(6, 2)$ **$\left(3\frac{1}{2}, 3\right)$**

4 EXAMPLE Error Prevention

Students may think that $\overline{ST}$ and $\overline{UW}$ meet at the centroid or orthocenter of $\triangle VSU$. Point out that since $\overline{ST}$ is an altitude and $\overline{UW}$ is a median, their point of intersection cannot be categorized.

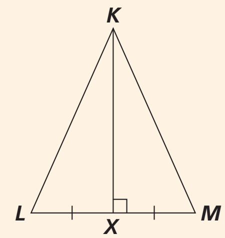
PowerPoint Additional Examples

3 M is the centroid of $\triangle WOR$, and $WM = 16$. Find WX.



24

4 Is $\overline{KX}$ a median, an altitude, neither, or both?

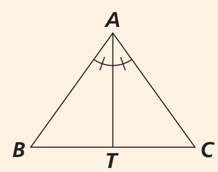


both

Resources

- Daily Notetaking Guide 5-3 L3
- Daily Notetaking Guide 5-3—Adapted Instruction L1

Closure



Use the diagram above to explain why the following must be true: The bisector of the vertex angle of an isosceles triangle is both an altitude and a median. **The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base by Theorem 4-5. Because the bisector is perpendicular, it is an altitude. Because it bisects the opposite side, it is a median.**

3. Practice

Assignment Guide

1 A B 1-10, 17-19, 21, 24, 29-31

2 A B 11-16, 20, 22, 23, 25-28, 32

C Challenge 33-36

Test Prep 37-41
Mixed Review 42-51

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 3, 12, 24, 28, 29.

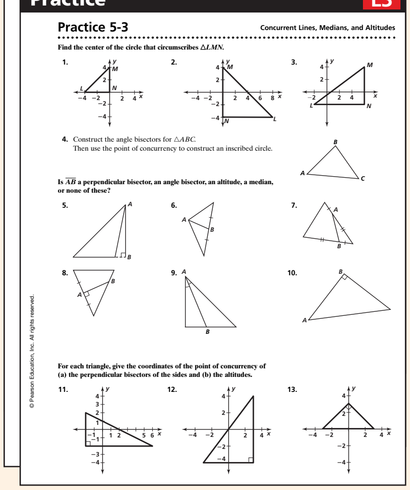
Alternative Method

Exercise 2 Students may trace and cut out the triangle and use paper folding, or carefully construct the perpendicular bisectors on graph paper, to find the point of intersection.

Exercises 3–7 If students use graph paper to draw the triangles, they will easily find the horizontal and vertical perpendicular bisectors.

Differentiated Instruction Resources

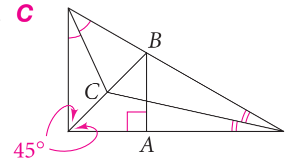
GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Adapted Practice L1
Practice L3



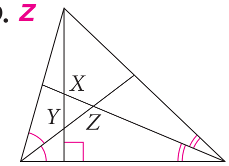
Example 2 (page 274)

Name the point of concurrency of the angle bisectors.

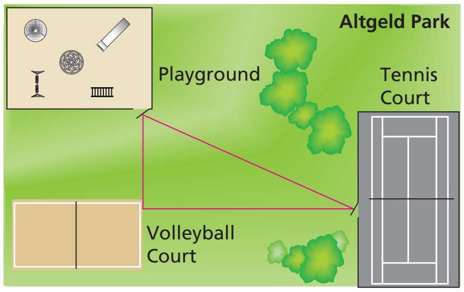
8. *C*



9. *Z*



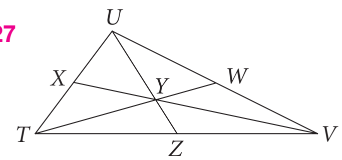
10. City Planning Copy the diagram of Altgeld Park. Show where park officials should place a drinking fountain so that it is equidistant from the tennis court, the playground, and the volleyball court.



Find the ⊥ bisectors of the sides of the △ formed by the tennis court, the playground, and the volleyball court. That point will be equidistant from the vertices of the △.

Example 3 (page 274)

In $\triangle TUV$, Y is the centroid.



11. If $YW = 9$, find TY and TW. $TY = 18$; $TW = 27$

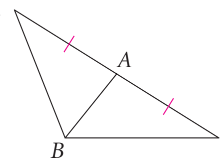
12. If $YU = 9$, find ZY and ZU. $ZY = 4\frac{1}{2}$; $ZU = 13\frac{1}{2}$

13. If $VX = 9$, find VY and YX. $VY = 6$; $YX = 3$

Example 4 (page 275)

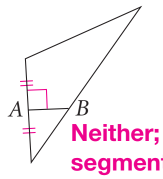
Is $\overline{AB}$ a median, an altitude, or neither? Explain.

14.



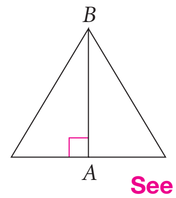
Median; A is a midpt.

15.



Neither; it's not a segment drawn from a vertex.

16.



See left.

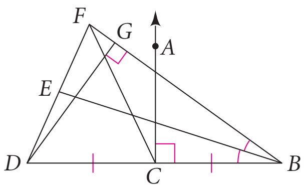
16. Altitude; $\overline{AB}$ is a segment drawn from a vertex of a △ perp. to the opp. side.

B Apply Your Skills

Constructions **Draw the triangle. Then construct the inscribed circle and the circumscribed circle.** 17–18. See margin.

17. right triangle, $\triangle DEF$ **18.** obtuse triangle, $\triangle STU$

In Exercises 19–22, name each figure in $\triangle BDF$.



19. an angle bisector $\overline{BE}$

20. a median $\overline{FC}$

21. a perpendicular bisector $\overrightarrow{CA}$

22. an altitude $\overline{DG}$

23. Critical Thinking A centroid separates a median into two segments. What is the ratio of the lengths of those segments? 1 : 2 or 2 : 1

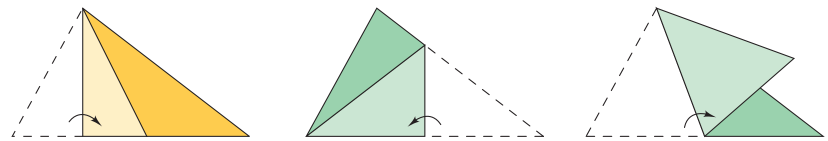
24. Writing Ivars found a yellowed parchment inside an antique book. It read:

From the spot I buried Olaf's treasure, equal sets of paces did I measure; each of three directions in a line, there to plant a seedling Norway pine. I could not return for failing health; now the hounds of Haiti guard my wealth.—Karl

After searching Caribbean islands for five years, Ivars found one with three tall Norway pines. How might Ivars find where Karl buried Olaf's treasure?

24. Find the circumcenter of the triangle formed by the three pines.

The figures below show how to construct medians and altitudes by paper folding.



To find an altitude, fold the triangle so that a side overlaps itself and the fold contains the opposite vertex.

To find a median, fold one vertex to another vertex. This locates the midpoint of a side.

Then fold so that the fold contains the midpoint and the opposite vertex.

Problem Solving Hint

Paper-folding an altitude is the same as paper-folding the perpendicular to a line through a point not on the line.

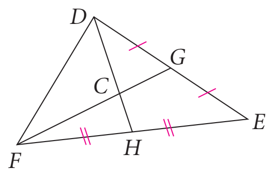
25–26. Check students' work.

25. Cut out a large triangle. Paper-fold very carefully to construct the three medians of the triangle and demonstrate Theorem 5-8.

26. Cut out a large acute triangle. Paper-fold very carefully to construct the three altitudes of the triangle and demonstrate Theorem 5-9.

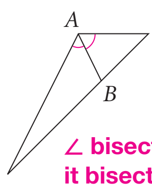
27. **Multiple Choice** C is the centroid of $\triangle DEF$. If $GF = 6x^2 + 9y$, what expression represents CF? **D**

Ⓐ $2x^2 + 9y$ Ⓑ $2x^2 + 3y$
Ⓒ $6x^2 + 9y$ Ⓓ $4x^2 + 6y$



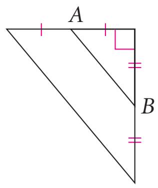
28. Is $\overline{AB}$ a perpendicular bisector, an angle bisector, a median, an altitude, or none of these? Explain. **b. None of these; it is a midsegment.**

a.

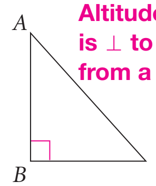


∠ bisector; it bisects an ∠.

b.



c.

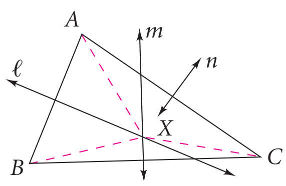


Altitude; $\overline{AB}$ is ⊥ to a side from a vertex.

29. **Developing Proof** Complete this proof of Theorem 5-6 by filling in the blanks. (GPS)

Given: Lines ℓ, m, and n are perpendicular bisectors of the sides of $\triangle ABC$. X is the intersection of lines ℓ and m.

Prove: Line n contains point X, and $XA = XB = XC$.



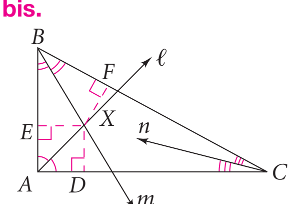
Proof: Since ℓ is the perpendicular bisector of **a.** __?__ **$\overline{AB}$**, $XA = XB$. Since m is the perpendicular bisector of **b.** __?__, $XB =$ **c.** __?__. Thus $XA = XB = XC$. Since $XA = XC$, X is on line n by the Converse of the **d.** __?__ Theorem.

b. $\overline{BC}$ c. XC d. ⊥ bis.

Proof **30.** Prove Theorem 5-7.

Given: Rays ℓ, m, and n are bisectors of the angles of $\triangle ABC$. X is the intersection of rays ℓ and m and $\overline{XD} \perp \overline{AC}$, $\overline{XE} \perp \overline{AB}$, $\overline{XF} \perp \overline{BC}$.

Prove: Ray n contains point X, and $XD = XE = XF$.



30. It is given that X is on line ℓ and line m. By the ∠ Bisect. Thm., $XD = XE$ and $XE = XF$. By the Trans. Prop. of =, $XD = XE = XF$. X is on ray n by the Conv. of the ∠ Bis. Thm.

31. What kind of triangle has its circumcenter on one of its sides? Explain.
A right triangle; check students' explanations.

GO nline Homework Help
Visit: PHSchool.com
Web Code: aue-0503

Error Prevention!

Exercise 10 Students may not realize that the playground and courts locate points. Discuss as a class why these particular points on the playground and courts might have been chosen.

Exercise 15 Point out that $\overline{AB}$ meets only half the conditions to be an altitude and only half the conditions to be a median, which means that it is neither.

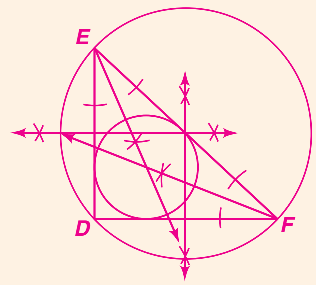
Exercise 27 Watch for students who think CF is $\frac{1}{3}GF$ instead of $\frac{2}{3}GF$. Ask: *Is CF larger or smaller than GC?* **larger**

Exercises 29, 30 Because using properties from two segments to prove concurrence is a new and sophisticated idea, discuss these proofs as a class after students complete them. Encourage students to ask questions about the strategy chosen for each proof.

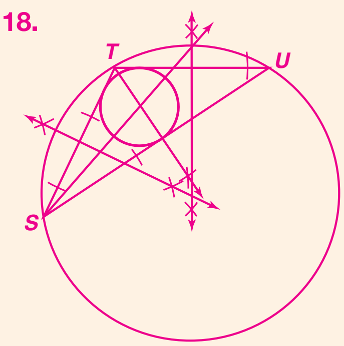
Connection to Discrete Math

Exercise 36 Euler (pronounced "oiler") is also responsible for the Seven Bridges of Königsberg problem, the proof of which was fundamental to the development of graph theory. Have students research Euler's contributions to mathematics.

17.



18.



4. Assess & Reteach

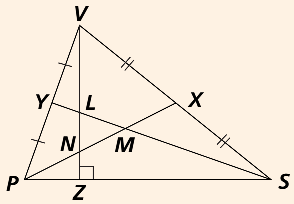
PowerPoint

Lesson Quiz

1. Complete the sentence:
To find the centroid of a triangle, you need to draw at least __?__ median(s). **two**

2. $\triangle FGH$ has vertices $F(-1, 2)$, $G(9, 2)$, and $H(9, 0)$. Find the center of the circle that circumscribes $\triangle FGH$. **(4, 1)**

Use the diagram for Exercises 3–5.



3. Identify all medians and altitudes drawn in $\triangle PSV$. **$\overline{PX}$ and $\overline{SY}$ are medians; $\overline{VZ}$ is an altitude.**

4. If $SY = 15$, find SM and MY. **$SM = 10$ and $MY = 5$**

5. If $MX = 14$, find PM and PX. **$PM = 28$ and $PX = 42$**

Alternative Assessment

P is a point inside $\triangle ABC$. Have students work in pairs to write a full description of the properties of point P if it is the circumcenter, incenter, centroid, or orthocenter of $\triangle ABC$.

Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 301
- Test-Taking Strategies, p. 296
- Test-Taking Strategies with Transparencies

Problem Solving Hint
You can prove Theorem 5-8 for a general $\triangle ABC$ with coordinates $A(0, 0)$, $B(2b, 2d)$, and $C(2c, 0)$ by following the steps for the particular $\triangle ABC$ in Exercise 32.

Challenge

32b. $\overleftrightarrow{AM}$: $y = \frac{3}{5}x$; $\overleftrightarrow{BN}$: $y = -3x + 12$; $\overleftrightarrow{CL}$: $y = -\frac{3}{7}x + \frac{24}{7}$

32d. $-\frac{3}{7}\left(\frac{10}{3}\right) + \frac{24}{7} = -\frac{10}{7} + \frac{24}{7} = \frac{14}{7} = 2$

35. Answers may vary. Sample: Let $\triangle ABC$ be isosc. with base ∠s B and C. If AD bisects $\angle A$, then it is $\perp$ to $\overline{BC}$, and therefore the altitude from $\angle A$. So, $\overleftrightarrow{AD}$ contains the circumcenter, incenter, centroid, and orthocenter.

32. Coordinate Geometry Complete the following steps to locate the centroid.

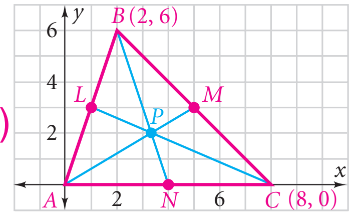
a. Find the coordinates of midpoints L, M, and N. **$L(1, 3)$; $M(5, 3)$; $N(4, 0)$**

b. Find equations of $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$, and $\overleftrightarrow{CL}$. **b. See below left.**

c. Find the coordinates of P, the intersection of $\overleftrightarrow{AM}$ and $\overleftrightarrow{BN}$. This is the centroid. **$\left(\frac{10}{3}, 2\right)$**

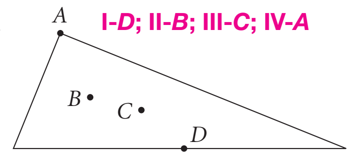
d. Show that point P is on $\overleftrightarrow{CL}$. **See left.**

e. Use the Distance Formula to show that point P is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side. **See margin.**

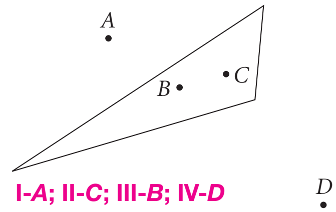


For Exercises 33 and 34, points of concurrency have been drawn for two triangles. Match the points with the lines and segments listed in I–IV.

33. **I-D; II-B; III-C; IV-A**



34. **I-A; II-C; III-B; IV-D**



I. perpendicular bisectors of sides
II. angle bisectors
III. medians
IV. lines containing altitudes

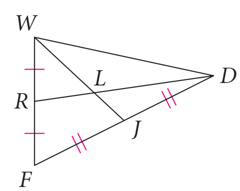
35. In an isosceles triangle, show that the circumcenter, incenter, centroid, and orthocenter can be four different points but all four must be collinear. **See left.**

36. History In 1765 Leonhard Euler proved that for any triangle, three of the four points of concurrency are collinear. The line that contains these three points is known as Euler's Line. Use Exercises 33 and 34 to determine which point of concurrency does not necessarily lie on Euler's Line. **∠ bisectors**

Test Prep

Multiple Choice

Use the figure at the right for Exercises 37–39.



37. What is RD if $RL = 54$ cm? **C**
A. 81 cm
B. 108 cm
C. 162 cm
D. 216 cm

38. What is WL if $WJ = 210$ mm? **H**
F. 70 mm
G. 105 mm
H. 140 mm
J. 157.5 mm

39. What is x if $WL = 15x$ and $LJ = 5x + 3$? **D**
A. 0.3
B. 0.4
C. 0.6
D. 1.2

Short Response

40. Name all types of triangles for which the centroid, circumcenter, incenter, and orthocenter are all inside the triangle. Classify the triangles according to the sides as well as the angles. **See margin.**

Extended Response

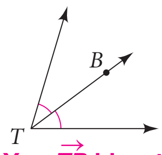
41. The point of concurrency of the three altitudes of a triangle lies outside the triangle. Where are its circumcenter, incenter, and centroid located in relation to the triangle? Draw and label a diagram to support each of your answers. **See back of book.**

Mixed Review

GO for Help

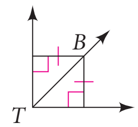
Lesson 5-2 **Determine whether point B must be on the bisector of $\angle T$. Explain.**

42.



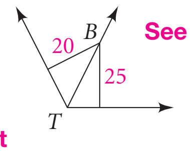
Yes; $\overrightarrow{TB}$ bisects the $\angle$.

43.



Yes; point B is equidistant from the sides.

44.



See left.

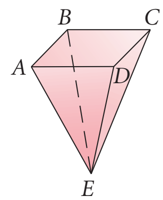
44. No; point B is not necessarily equidistant from the sides.

Lesson 3-4 **Classify each $\triangle JKL$ by its angles.**

45. $m\angle J = 37, m\angle K = 53, m\angle L = 90$ right

46. $m\angle J = 47, m\angle K = 98, m\angle L = 35$ obtuse

Lesson 1-4 **In the figure at the right, $ABCD$ is a square. Identify each of the following.** 47–51. Answers may vary.



47. a line skew to $\overleftrightarrow{ED}$ $\overleftrightarrow{AB}$

48. a line skew to $\overleftrightarrow{EB}$ $\overleftrightarrow{AD}$

49. two intersecting planes

50. two parallel segments

51. the intersection of plane ABC and plane BCE $\overleftrightarrow{BC}$

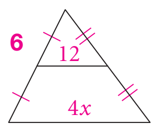
49. ABC and ADE

50. $\overline{AB}$ and $\overline{CD}$

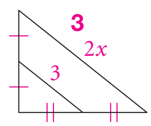
Checkpoint Quiz 1 — Lessons 5-1 through 5-3

x^2 **Algebra** **Find the value of x.**

1.

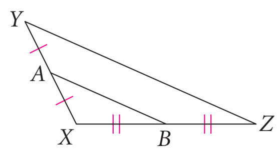


2.



3. a. $\overline{AB}$ is a midsegment of $\triangle XYZ$. $AB = 52$. Find YZ. 104

b. $AX = 26$ and $BZ = 36$. Find the perimeter of $\triangle XYZ$. 228

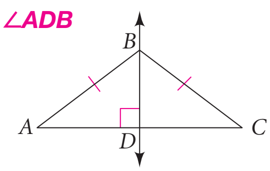


Use the diagram. What can you conclude about each of the following? Explain.

4. $\angle CDB$ right $\angle$; supp. to $\angle ADB$

5. $\triangle ABD$ and $\triangle CBD$

6. $\overline{AD}$ and $\overline{DC}$
$\overline{AD} \cong \overline{DC}$; CPCTC

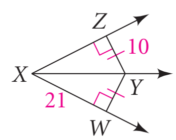


5. $\triangle ABD \cong \triangle CBD$; HL

Use the figure at the right. 7–8. See left.

7. What can you conclude about $\overrightarrow{XY}$? Explain.

8. Find XZ. Justify your response.



7. $\overrightarrow{XY}$ bisects $\angle ZXW$; Y is equidist. from $\overrightarrow{XZ}$ and $\overrightarrow{XW}$.

8. 21; $\triangle XYZ \cong \triangle XYW$ by HL, so $XZ = 21$ by CPCTC.

Writing **For a given triangle, describe how you can construct the following.**

9. a median 9–10. See margin.

10. an altitude

Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 5-1 through 5-3.

Resources

Grab & Go

- Checkpoint Quiz 1

32. e. $AM = \sqrt{34}$; $AP = \sqrt{\frac{136}{9}} = \frac{2}{3}\sqrt{34}$; $BN = \sqrt{40} = 2\sqrt{10}$; $BP = \sqrt{\frac{160}{9}} = \frac{4}{3}\sqrt{10}$; $CL = \sqrt{58}$; $CP = \sqrt{\frac{232}{9}} = \frac{2}{3}\sqrt{58}$

40. [2] any acute $\triangle$; or a list that contains all of the following: equiangular $\triangle$s, equilateral $\triangle$s, acute isosceles $\triangle$s, acute scalene $\triangle$

[1] a list that does not contain equiangular $\triangle$s, equilateral $\triangle$s, acute isosceles $\triangle$s, or scalene $\triangle$

Checkpoint Quiz

9. Answers may vary. Sample: Bisect a side of a $\triangle$. Connect the opp. vertex with the midpt.

10. Use the procedure for constructing a $\perp$ to a line from a point not on the line.