

Proportions in Triangles

1. Plan

Objectives

- To use the Side-Splitter Theorem
- To use the Triangle-Angle-Bisector Theorem

Examples

- Using the Side-Splitter Theorem
- Real-World Connection
- Using the Triangle-Angle-Bisector Theorem



Math Background

The Side-Splitter Theorem represents a generalization of the Triangle Midsegment Theorem from Chapter 5. The concept of similarity makes possible this generalization. The Side-Splitter Theorem applied to three parallel lines proves the Triangle-Angle Bisector Theorem.

More Math Background: p. 364D

Lesson Planning and Resources

See p. 364E for a list of the resources that support this lesson.

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Using Similar Figures

Lesson 7-2: Example 3

Extra Skills, Word Problems, Proof Practice, Ch. 7

What You'll Learn

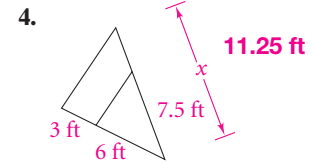
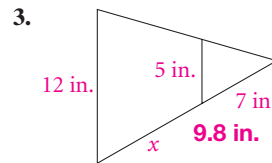
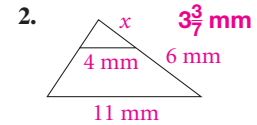
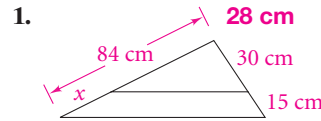
- To use the Side-Splitter Theorem
- To use the Triangle-Angle-Bisector Theorem

... And Why

To design a sail, as in Example 2

Check Skills You'll Need

The two triangles in each diagram are similar. Find the value of x in each.



1

Using the Side-Splitter Theorem

You can use similar triangles to prove the following theorem.



Key Concepts

Theorem 7-4

Side-Splitter Theorem

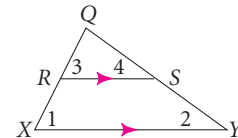
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

Proof

Proof of Theorem 7-4

Given: $\triangle QXY$ with $\overleftrightarrow{RS} \parallel \overleftrightarrow{XY}$

Prove: $\frac{XR}{RQ} = \frac{YS}{SQ}$



Test-Taking Tip

Whenever you see a line that passes through a triangle and is parallel to one of the sides, the Side-Splitter Theorem may apply.

Statements

Reasons

- $\overleftrightarrow{RS} \parallel \overleftrightarrow{XY}$
- $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$
- $\triangle QXY \sim \triangle QRS$
- $\frac{XQ}{RQ} = \frac{YQ}{SQ}$
- $XQ = XR + RQ$, $YQ = YS + SQ$
- $\frac{XR + RQ}{RQ} = \frac{YS + SQ}{SQ}$
- $\frac{XR}{RQ} = \frac{YS}{SQ}$

- Given
- If lines are \parallel , then corr. \angle s are \cong .
- AA \sim Postulate
- Corr. sides of $\sim \triangle$ s are proportional.
- Segment Addition Postulate
- Substitute.
- A Property of Proportions

Differentiated Instruction Solutions for All Learners

Special Needs L1

Have students draw three parallel lines cut by two transversals. Students use a ruler to measure the segments intercepted on the transversals and observe they are proportional, and not congruent.

learning style: tactile

Below Level L2

Review the properties of proportions, especially the Cross-Product Property, before students read the proof of Theorem 7-4 and work through the examples.

learning style: verbal

2. Teach

1 EXAMPLE Using the Side-Splitter Theorem

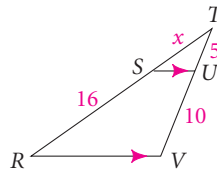
Gridded Response Find the value of x .

$$\frac{TS}{SR} = \frac{TU}{UV} \quad \text{Side-Splitter Theorem}$$

$$\frac{x}{16} = \frac{5}{10} \quad \text{Substitute.}$$

$$x = \frac{5}{10} \cdot 16 \quad \text{Solve for } x.$$

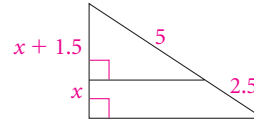
$$x = 8$$



	7	7	8
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9



- 1 Use the Side-Splitter Theorem to find the value of x . **1.5**



The following corollary to the Side Splitter Theorem says that parallel lines divide all transversals proportionally. You will prove this corollary in Exercise 35.

Guided Instruction

Teaching Tip

Discuss as a class the proof of Theorem 7-4. In particular, have students use algebra to show how step 7 follows from step 6.

1 EXAMPLE Alternative Method

ST can be found without using the Side-Splitter Theorem by remembering that sides of similar triangles are proportional. Ask: *What proportion could you write and solve?* $\frac{ST}{ST + 16} = \frac{5}{15}$

2 EXAMPLE Diversity

Some students may be unaware of the different shapes of sails for ancient and modern boats. If possible, show pictures of boats with different kinds of sails.

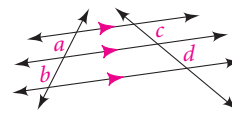


Key Concepts

Corollary Corollary to Theorem 7-4

If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

$$\frac{a}{b} = \frac{c}{d}$$



Real-World Connection

You windsurf with a large sail in light winds and a small sail in strong winds.

2 EXAMPLE Real-World Connection

Sail Making Sail makers sometimes use a computer to create a pattern for a sail. After they cut out the panels of the sail, they sew them together to form the sail.

The edges of the panels in the sail at the right are parallel. Find the lengths x and y .

$$\frac{2}{x} = \frac{1.7}{1.7} \quad \text{Side-Splitter Theorem}$$

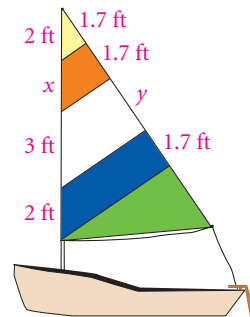
$$x = 2$$

$$\frac{3}{2} = \frac{y}{1.7} \quad \text{Corollary to the Side-Splitter Theorem}$$

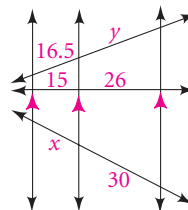
$$\frac{3}{2}(1.7) = y \quad \text{Solve for } y.$$

$$2.55 = y$$

- Length x is 2 ft and length y is 2.55 ft.

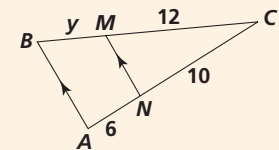


- 2 Solve for x and y .
 $x = \frac{225}{13}$; $y = 28.6$



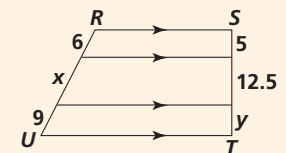
Additional Examples

- 1 Find y .



7.2

- 2 The segments joining the sides of trapezoid $RSTU$ are parallel to its bases. Find x and y .



$x = 15$, $y = 7.5$

Advanced Learners L4

Use the Side-Splitter Theorem to prove that, if a line parallel to one side of a triangle intersects the midpoint of another side, then it intersects the midpoint of the third side.

learning style: verbal

English Language Learners ELL

For Example 2, review the terms *sail maker* and *panels*. Make sure students understand the pieces of the sail are cut and sewn so the bases (edges) of the pieces are parallel.

learning style: verbal

Guided Instruction

Math Tip

After introducing the Triangle-Angle-Bisector Theorem, remind students of the Angle Bisector Theorem in Lesson 5-2. Have them explain how the theorems differ.

Technology Tip

Students can model the proof of Theorem 7-5 using geometry software.

3 EXAMPLE Error Prevention

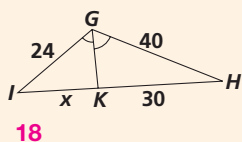
Some students looking at the diagram for this example may think they can use the proportions from Lesson 7-4 that apply only to right triangles. Ask: *What must be true to apply the theorems and corollaries from Lesson 7-4?*

The triangle must be a right triangle with an altitude to the hypotenuse.

PowerPoint

Additional Examples

3 Find the value of x .

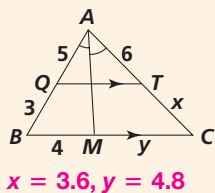


Resources

- Daily Notetaking Guide 7-5 **L3**
- Daily Notetaking Guide 7-5—Adapted Instruction **L1**

Closure

In $\triangle ABC$, $\overline{QT} \parallel \overline{BC}$ and \overline{AM} bisects $\angle BAC$. Find x and y .



2

Using the Triangle-Angle-Bisector Theorem

You can use the Side-Splitter Theorem to prove the following relationship.



Key Concepts

Theorem 7-5

Triangle-Angle-Bisector Theorem

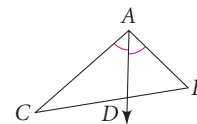
If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

Proof

Proof of Theorem 7-5

Given: $\triangle ABC$, \overline{AD} bisects $\angle CAB$.

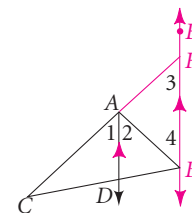
Prove: $\frac{CD}{DB} = \frac{CA}{BA}$



Draw $\overline{BE} \parallel \overline{DA}$. Extend \overline{CA} to meet \overline{BE} at point F .

Proof: By the Side-Splitter Theorem, $\frac{CD}{DB} = \frac{CA}{AF}$.

By the Corresponding Angles Postulate, $\angle 3 \cong \angle 1$. Since \overline{AD} bisects $\angle CAB$, $\angle 1 \cong \angle 2$. By the Alternate Interior Angles Theorem, $\angle 2 \cong \angle 4$. Using the Transitive Property of Congruence, you know that $\angle 3 \cong \angle 4$. By the Converse of the Isosceles Triangle Theorem, $BA = AF$. Substituting BA for AF , $\frac{CD}{DB} = \frac{CA}{BA}$.



Problem Solving Hint

Drawing $\overline{BE} \parallel \overline{DA}$ sets up $\triangle BCF$ for the Side-Splitter Theorem, as well as congruent $\triangle 1, 2, 3$, and 4 .

3 EXAMPLE Using the Triangle-Angle-Bisector Theorem

Algebra Find the value of x .

$$\frac{PS}{SR} = \frac{PQ}{RQ}$$

Triangle-Angle-Bisector Theorem

$$\frac{x}{6} = \frac{8}{5}$$

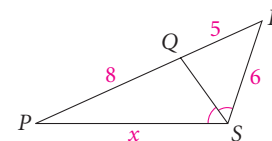
Substitute.

$$5x = 48$$

Cross-Product Property

$$x = 9.6$$

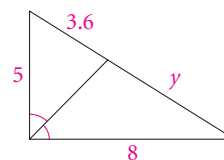
Solve for x .



Quick Check

3 Find the value of y .

5.76



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice*.

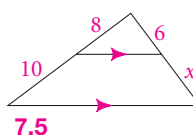
Practice and Problem Solving

A Practice by Example x^2 **Algebra** Solve for x .

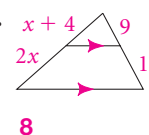


Example 1
(page 399)

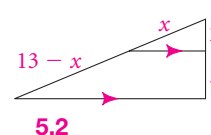
1.



2.



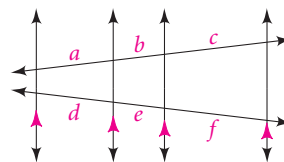
3.



Example 2
(page 399)

Use the figure at the right to complete each proportion.

4. $\frac{a}{b} = \frac{\square}{e} d$ 5. $\frac{b}{\square} = \frac{e}{f} c$
 6. $\frac{f}{e} = \frac{c}{\square} b$ 7. $\frac{a}{b+c} = \frac{\square}{e+f} d$



x² Algebra Solve for x.

8. 9. 10.

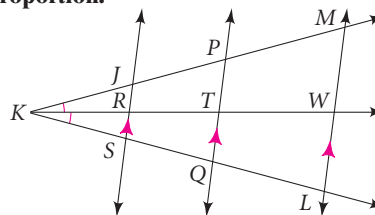
Example 3
(page 400)

x² Algebra Solve for x.

11. 12. 13. 14. 15. 16.

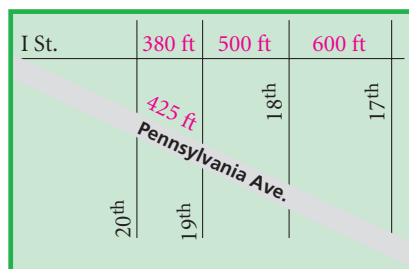
Use the figure at the right to complete each proportion.

17. $\frac{RS}{\square} = \frac{JR}{KJ} KS$ 18. $\frac{KJ}{JP} = \frac{KS}{\square} SQ$
 19. $\frac{QL}{PM} = \frac{SQ}{\square} JP$ 20. $\frac{PT}{\square} = \frac{TQ}{KQ} KP$
 21. $\frac{KL}{LW} = \frac{\square}{MW} KM$ 22. $\frac{\square}{KP} = \frac{LQ}{KQ} PM$
 23. $\frac{\square}{SQ} = \frac{JK}{KS} JP$ 24. $\frac{KL}{KM} = \frac{\square}{MW} LW$



B Apply Your Skills

Urban Design In Washington, D.C., 17th, 18th, 19th, and 20th Streets are parallel streets that intersect Pennsylvania Avenue and I Street.



25. How long (to the nearest foot) is Pennsylvania Avenue between 19th Street and 18th Street? **559 ft**
26. How long (to the nearest foot) is Pennsylvania Avenue between 18th Street and 17th Street? **671 ft**
27. The sides of a triangle are 5 cm, 12 cm, and 13 cm long. Find the lengths, to the nearest tenth, of the segments into which the bisector of each angle divides the opposite side. **2.4 cm and 2.6 cm; 3.3 cm and 8.7 cm; 3.8 cm and 9.2 cm**
28. **Open-Ended** In a triangle, the bisector of an angle divides the opposite side into two segments with lengths 6 cm and 9 cm. How long could the other two sides of the triangle be? (*Caution:* Make sure the three sides satisfy the Triangle Inequality Theorem.) **Answers may vary. Sample: 9 cm and 13.5 cm**



Real-World Connection

Careers Some urban planners specialize in environmental issues or historic preservation.

3. Practice

Assignment Guide

1 A B 1-10, 25-28, 31, 33-39, 41-43

2 A B 11-24, 29, 30, 32, 40, 44

C Challenge 45-47

Test Prep 48-51

Mixed Review 52-65

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 8, 14, 26, 34, 36.

Connection to Algebra

Exercise 3 Use this exercise to check how well students solve complicated proportions. Review solving equations with variables on both sides.

Exercises 14, 15 Have students discuss why the unlabeled segment has length 6 - x in Exercise 14 and 10 - x in Exercise 15.

Differentiated Instruction Resources

GPS Guided Problem Solving L3

Enrichment L4

Reteaching L2

Adapted Practice L1

Practice L3

Practice 7-5 Areas of Regular Polygons

Find the values of the variables for each regular polygon. Leave your answers in simplest radical form.

1. 2. 3.

Each regular polygon has radii and an apothem as shown. Find the measure of each numbered angle.

4. 5. 6.

Find the area of each equilateral triangle, given the radius. Leave your answers in simplest radical form.

7. 8. 9.

Find the area of each regular polygon to the nearest square inch.

10. 11. 12.

Careers

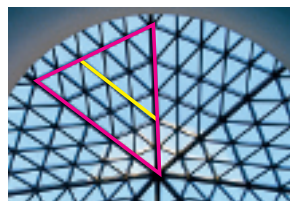
Exercise 29 Surveyors use methods of indirect measurement that are based on concepts found in Chapter 7. Encourage students to find out what math courses are necessary for a career in surveying.

Exercise 30 This suggests the theorem *If an angle bisector of a triangle bisects the opposite side, then the triangle is isosceles.* There was no way for students to prove this until they learned the Triangle-Angle-Bisector Theorem.

Exercises 31–33 You may need to review how to solve quadratic equations.

Connection to Environmental Science

Exercise 39 As industrial production increased dramatically during the twentieth century, world leaders and citizens showed increasing concern about the environment. Oil spills contaminate water, kill plants and animals, and ultimately affect the environment far from the actual spill site.



Real-World Connection

In this glass roof, parallel lines divide the sides of triangles proportionally.

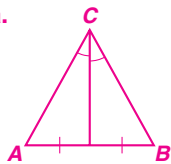
Visual Learners

Exercise 40 Some students may find spatial visualization difficult. Provide a model for them to examine.

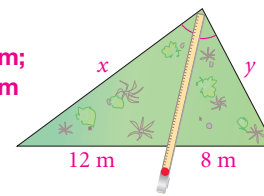
GO for Help

For a guide to solving Exercise 29, see p. 405.

30a.

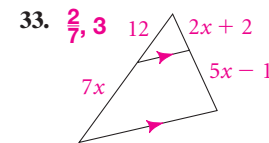
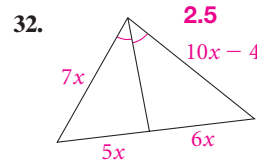
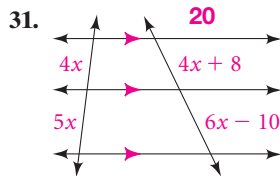


29. **Surveying** The perimeter of the triangular lot at the right is 50 m. The surveyor's tape $x = 18$ m; bisects an angle. Find the lengths x and y . $y = 12$ m



30. **Critical Thinking** Sharell draws $\triangle ABC$. She finds that the bisector of $\angle C$ bisects the opposite side.
- Sketch $\triangle ABC$ and the bisector. **See left.**
 - Writing** What type of triangle is $\triangle ABC$? Explain your reasoning. **isosceles; Δ - \angle Bisector Thm.**

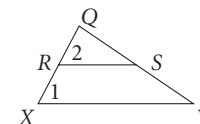
Algebra Solve for x .



34. **Developing Proof** Copy and complete this two-column proof of the Converse of the Side-Splitter Theorem: If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

Given: $\frac{XR}{RQ} = \frac{YS}{SQ}$

Prove: $\overline{RS} \parallel \overline{XY}$



Statements	Reasons
1. $\frac{XR}{RQ} = \frac{YS}{SQ}$	a. ? Given
2. $\frac{XR + RQ}{RQ} = \frac{YS + SQ}{SQ}$	b. ? Prop. of Proportions
3. $\frac{XQ}{RQ} = \frac{YQ}{SQ}$	c. ? Segment Add. Post.
4. $\angle Q \cong \angle Q$	d. ? Reflexive Prop. of \cong
5. $\triangle XQY \sim \triangle RQS$	e. ? SAS \sim Thm.
6. $\angle 1 \cong \angle 2$	f. ? Corr. \angle of $\sim \Delta$ are \cong.
7. $\overline{RS} \parallel \overline{XY}$	g. ? If corr. Δ are \cong, lines \parallel.

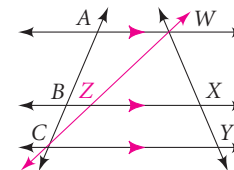
- Proof** 35. Follow the steps below. Write a proof of the Corollary to the Side-Splitter Theorem found on page 399.

Given: $\overleftrightarrow{AW} \parallel \overleftrightarrow{BX} \parallel \overleftrightarrow{CY}$

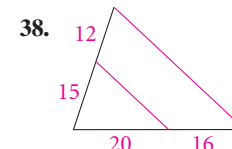
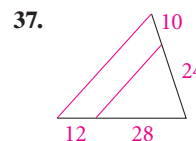
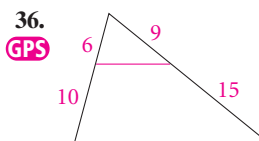
Prove: $\frac{AB}{BC} = \frac{WX}{XY}$

Begin by drawing \overleftrightarrow{WC} , intersecting \overleftrightarrow{BX} at point Z.

- Apply the Side-Splitter Theorem to $\triangle ACW$: $\frac{AB}{BC} = \frac{WZ}{ZC}$. $\frac{AB}{BC}$
- Apply the Side-Splitter Theorem to $\triangle CWY$: $\frac{WZ}{ZC} = \frac{WX}{XY}$. $\frac{WX}{XY}$
- Substitute to prove the corollary. $\frac{AB}{BC} = \frac{WX}{XY}$



Determine whether the red segments are parallel. Explain each answer. You can use the theorem proved in Exercise 34. 36–38. See margin.



36. **Yes;** since $\frac{6}{10} = \frac{9}{15}$, the segments are \parallel by the Converse of the Side-Splitter Thm.

37. **No;** $\frac{28}{12} \neq \frac{24}{10}$.

38. **Yes;** since $\frac{15}{12} = \frac{20}{16}$, the segments are \parallel by the Converse of the Side-Splitter Thm.

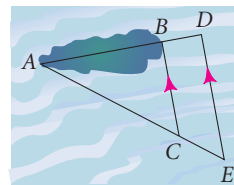
GO Online Homework Help
Visit: PHSchool.com
Web Code: aue-0705



Real-World Connection

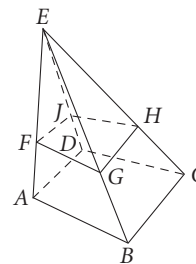
You measure an oil spill to find the size of the boom you'll need to contain it.

39. **Oil Spills** Describe how you could use the figure at the right to find the length of the oil spill indirectly. What measurements and calculations would you use? **See margin.**
40. An angle bisector of a triangle divides the opposite side of the triangle into segments 5 cm and 3 cm long. A second side of the triangle is 7.5 cm long. Find all possible lengths for the third side of the triangle. **4.5 cm or 12.5 cm**



Geometry in 3 Dimensions In the figure at the right, $\overleftrightarrow{FG} \parallel \overleftrightarrow{AB}$, $\overleftrightarrow{GH} \parallel \overleftrightarrow{BC}$, $AF = 2$, $FE = 4$, and $BG = 3$.

41. Find GE . **6**
42. If $EH = 5$, find HC . **2.5**
43. If $FG = 3$, find the perimeter of $\triangle ABE$. **19.5**



- Proof** 44. One side of a triangle is k times as long as a second side. The bisector of their angle cuts the third side into two segments. Prove that one of those segments is k times as long as the other.
- Challenge Proof** 45. Use the definition in part (a) to prove the statements in parts (b) and (c).
- Write a definition for a midsegment of a parallelogram.
 - A parallelogram midsegment is parallel to two sides of the parallelogram.
 - A parallelogram midsegment bisects the diagonals of a parallelogram. **a–c. See back of book.**
46. State the converse of the Triangle-Angle-Bisector Theorem. Give a convincing argument that the converse is true or a counterexample to prove that it is false. **See back of book.**
47. In $\triangle ABC$, the bisectors of $\angle A$, $\angle B$, and $\angle C$ cut the opposite sides into lengths a_1 and a_2 , b_1 and b_2 , and c_1 and c_2 , respectively, labeled in this order counterclockwise around $\triangle ABC$. Find the perimeter of $\triangle ABC$ for each of the following.
- $a_1 = \frac{5}{3}$, $a_2 = \frac{10}{3}$, $b_1 = \frac{15}{4}$ **14**
 - $a_1 = \frac{10}{3}$, $b_1 = \frac{10}{9}$, $c_1 = \frac{8}{7}$ **11**

44. The two segments have lengths x and y .

$$\frac{x}{y} = \frac{ks}{s} = k, \text{ so } x = ky.$$



Test Prep

Multiple Choice

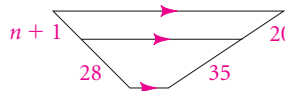
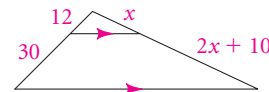
50. [2] $\frac{n+1}{28} = \frac{20}{35}$;
 $35n + 35 = 560$;
 $n = 15$

[1] correct proportion solved incorrectly

Short Response

Extended Response

48. Use the figure at the right. What is x ? **D**
- A. 5 B. 10
 C. 15 D. 20
49. The legs of a right triangle have lengths 7 and 24. The bisector of the right angle divides the hypotenuse into two segments. What is the length of the shorter segment of the hypotenuse to the nearest tenth? **F**
- F. 5.6 G. 8.0 H. 19.4 J. 25
50. What is n ? Show your work. **See left.**
51. The bisectors of an angle of a triangle divide the opposite side of the triangle into segments 4 cm and 5 cm long. A second side of the triangle is 6 cm long.
- Draw two diagrams you can use to find the two possible different lengths for the third side. **a–b. See back of book.**
 - Use each diagram in part (a) to write a proportion. Solve for each possible length of the third side of the triangle. Show your work.



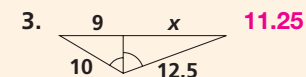
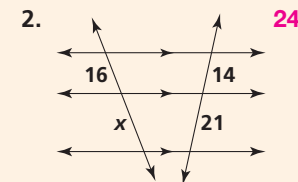
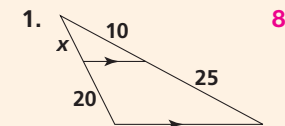
39. Measure \overline{AC} , \overline{CE} , and \overline{BD} . Use the Side-Splitter Thm. Write the proport. $\frac{AC}{CE} = \frac{AB}{BD}$ and solve for AB .

4. Assess & Reteach

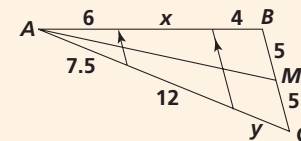
PowerPoint

Lesson Quiz

Solve for x in each diagram.



Use the diagram below for Exercises 4–6.



4. Find x . **9.6**
5. Find y . **5**
6. Explain how you know that \overline{AM} is not the angle bisector of $\angle BAC$. **If it were, $\frac{AB}{AC} = \frac{BM}{MC} = 1$. But $AC > AB$.**

Alternative Assessment

Have each student draw two triangles, construct a segment parallel to a side on one triangle and an angle bisector on the other triangle, and then use rulers and calculators to illustrate the Side-Splitter and Triangle-Angle-Bisector Theorems.

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 411
- Test-Taking Strategies, p. 406
- Test-Taking Strategies with Transparencies

Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 7-3 through 7-5.

Resources

Grab & Go

- Checkpoint Quiz 2

Mixed Review



Lesson 7-4

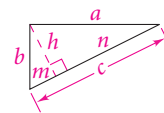
Refer to the figure to complete each proportion.

$$52. \frac{n}{h} = \frac{h}{m} m$$

$$53. \frac{m}{b} = \frac{b}{c} m$$

$$54. \frac{n}{a} = \frac{a}{m} c$$

$$55. \frac{m}{h} = \frac{m}{n} h$$



Lesson 6-4

x^2 Algebra $RSTV$ is a rectangle. Find the lengths of the diagonals \overline{RT} and \overline{SV} .

$$56. RT = SV = 38$$

$$56. RT = 5x + 8, SV = x + 32$$

$$57. RT = 42 - x, SV = 9x - 8$$

$$57. RT = SV = 37$$

$$58. RT = 8x - 4, SV = 6x + 9$$

$$RT = SV = 48$$

$$59. RT = 3x + 5, SV = 5x + 4$$

$$RT = SV = 6.5$$

Lesson 5-3

Find the center of the circle that you can circumscribe about each $\triangle ABC$.

$$60. A(0, 0) \quad (3, -3)$$

$$B(6, 0)$$

$$C(0, -6)$$

$$61. A(2, 5) \quad (0, 2)$$

$$B(-2, 5)$$

$$C(-2, -1)$$

$$62. A(-2, 0) \quad (1.5, 2.5)$$

$$B(5, 5)$$

$$C(-2, 5)$$

For $\triangle ABC$, name the point of intersection associated with each set of segments.

$$63. \text{the medians} \\ \text{centroid}$$

$$64. \text{the angle bisectors} \\ \text{incenter}$$

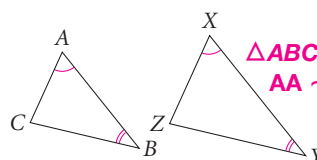
$$65. \text{the altitudes} \\ \text{orthocenter}$$

Checkpoint Quiz 2

Lessons 7-3 through 7-5

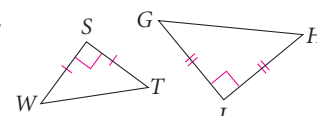
Determine whether the triangles are similar. If so, write the similarity statement. Also, write the postulate or theorem that proves they are similar.

1.



$$\triangle ABC \sim \triangle XYZ; \\ AA \sim \text{Post.}$$

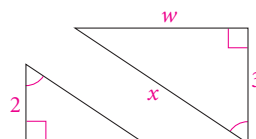
2.



$$\triangle WST \sim \triangle HJG; SAS \sim \text{Thm.}$$

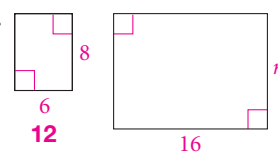
x^2 Algebra The polygons are similar. Find the value of each variable.

3.



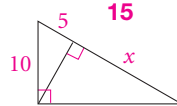
$$x = \frac{3\sqrt{13}}{2}; w = 4.5$$

4.

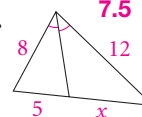


x^2 Algebra Find the value of each variable.

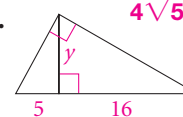
5.



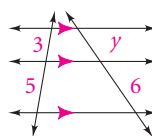
6.



7.

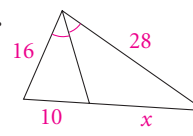


8.



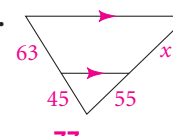
$$3.6$$

9.



$$17.5$$

10.



$$77$$