Chapter Review

Vocabulary Review

Cross-Product Property (p. 367)  
extended proportion (p. 367)  
geometric mean (p. 392)  
golden ratio (p. 375)  
golden rectangle (p. 375)  
indirect measurement (p. 384)  
proportion (p. 367)  
ratio (p. 366)  
scale (p. 368)  
scale drawing (p. 368)  
similar (p. 373)  
similarity ratio (p. 373)

Choose the correct term to complete each sentence.

1. Two polygons are __similar__ if corresponding angles are congruent and corresponding sides are proportional.

2. The __cross product__ states that the product of the extremes is equal to the product of the means.

3. A __golden rectangle__ is a rectangle that can be divided into a square and a rectangle that is similar to the original rectangle.

4. The ratio of the lengths of corresponding sides of two similar figures is the __similarity ratio__.

5. A __proportion__ is a statement that two ratios are equal.

6. Finding distances using similar triangles is called __indirect measurement__.

7. The length and width of a golden rectangle are in the __golden ratio__.

Skills and Concepts

7-1 Objectives

To write ratios and solve proportions

A ratio is a comparison of two quantities by division. You can write the ratio of \( a \) to \( b \) or \( a : b \) as the quotient \( \frac{a}{b} \) when \( b \neq 0 \).

A __proportion__ is a statement that two ratios are equal. According to the Properties of Proportions, \( \frac{a}{b} = \frac{c}{d} \) is equivalent to

\[
\begin{align*}
(1) \quad & ad = bc \\
(2) \quad & \frac{b}{d} = \frac{c}{a} \\
(3) \quad & \frac{a}{c} = \frac{b}{d} \\
(4) \quad & \frac{a}{b} + \frac{c}{d} = \frac{c}{a} + \frac{b}{d}
\end{align*}
\]

Property 1, above, illustrates the Cross-Product Property, which states that the product of the extremes is equal to the product of the means.

When three or more ratios are equal, you can write an extended proportion.

In a scale drawing, the scale compares each length in the drawing to the actual length being represented.

Dollhouses  Dollhouse furnishings come in different sizes depending on the size of the dollhouse. For each exercise, write a ratio of the size of the dollhouse item to the size of the larger item.

8. dollhouse sofa: \( \frac{1}{2} \) in. long;  
   real sofa: 6 ft long  
   \( \frac{1}{48} \)

9. dollhouse piano: \( \frac{3}{4} \) in. high   
   real piano: 3 ft 6 in. high  
   \( 1:24 \)

If \( \frac{p}{q} = \frac{2}{5} \), tell whether each equation must be true. Explain. 10–13. See margin.

10. \( 2q = 5p \)

11. \( \frac{5}{2} = \frac{q}{p} \)

12. \( 5q = 2p \)

13. \( \frac{p}{2} = \frac{q}{5} \)

10. True; use the Cross-Product Prop.

11. True; the cross product eq. is equivalent to the original proportion.

12. False; the cross product eq. is not equivalent to the original proportion.

13. True; the cross product eq. is equivalent to the original proportion.
Similar polygons have congruent corresponding angles and proportional corresponding sides. The ratio of the lengths of corresponding sides is the similarity ratio.

A golden rectangle is a rectangle that can be divided into a square and a rectangle that is similar to the original rectangle. In any golden rectangle, the length and the width are in the golden ratio, which is \( \frac{1 + \sqrt{5}}{2} \approx 1.618 : 1 \).

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar by the Angle-Angle Similarity Postulate (AA ~).

If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar by the Side-Angle-Side Similarity Theorem (SAS ~).

If the corresponding sides of two triangles are proportional, then the triangles are similar by the Side-Side-Side Similarity Theorem (SSS ~).

Methods of indirect measurement use similar triangles and measurements to find distances that are difficult to measure directly.

14. If \( \triangle MNP \sim \triangle RST \), which angles are congruent? Write an extended proportion to indicate the proportional corresponding sides of the triangles.

15. Art. An artist is creating a stained glass window and wants it to be a golden rectangle. To the nearest inch, what should be the length if the width is 24 in.? 39 in. or 15 in.

The triangles are similar. Find the similarity ratio of the first to the second.

16. \( \angle M \cong \angle R, \angle N \cong \angle S, \angle P \cong \angle T; \frac{MP}{RS} = \frac{MP}{RT} = \frac{NP}{SY} \)

17. 2:3 or 5:3

18. Algebra. The polygons are similar. Find the value of each variable.

19. \( x = 12; y = 15 \)

Are the triangles similar? If so, write the similarity statement and name the postulate or theorem you used. If not, explain.

20. \( \triangle XYZ \sim \triangle JKL; \) SAS ~ Thm.

21. Not ~; Corr. sides are not prop.
22. Two right triangles have an acute angle with the same measure. Name the theorem or postulate that is the most direct way to prove the triangles similar. **AA ~ Post.**

23. **Indirect Measurement** A crate is 1.5 ft high and casts a 2-ft shadow. At the same time, an apple tree casts an 18-ft shadow. How tall is the tree? **13.5 ft**

The **geometric mean** of two positive numbers \(a\) and \(b\) is the positive number \(x\) such that:

\[
\text{when the altitude is drawn to the hypotenuse of a right triangle:}
\]

- the two triangles formed are similar to the original triangle and to each other;
- the length of the altitude is the geometric mean of the lengths of the segments of the hypotenuse; and
- the length of each leg is the geometric mean of the length of the adjacent hypotenuse segment and the length of the hypotenuse.

**Algebra** Find the geometric mean of each pair of numbers.

- 24. 4 and 25: **10**
- 25. 3 and 300: **30**
- 26. 5 and 12: **2\sqrt{15}**
- 27. \(\frac{1}{2}\) and 28: **\(\frac{2\sqrt{7}}{3}\)**
- 28. 0.36 and 4: **1.2**
- 29. **1.2**
- 30. **10**
- 31. **30**
- 32. **12**

**Algebra** Find the values of the variables. When an answer is not a whole number, leave it in simplest radical form. **\(x = 2\sqrt{21}; y = 4\sqrt{3}; z = 4\sqrt{7}\)**

- 30. \(x = 15; y = 12; z = 20\)
- 31. \(x = 2\sqrt{3}; y = 6; z = 4\sqrt{3}\)
- 32. \(x = 3\sqrt{2}; y = 4\sqrt{2}; z = 5\sqrt{3}\)

The **Side-Splitter Theorem** states that if a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally. If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

The **Triangle-Angle-Bisector Theorem** states that if a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

**Algebra** Find the value of \(x\).

- 33. **7.5**
- 34. **5.5**
- 35. **37.5**
- 36. **11.25**
- 37. **17.5**
- 38. **12**