8-1 The Pythagorean Theorem and Its Converse

What You'll Learn

- To use the Pythagorean Theorem
- To use the Converse of the Pythagorean Theorem

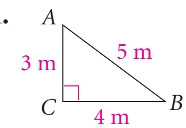
. . . And Why

To find the distance between two docks on a lake, as in Example 3

Check Skills You'll Need

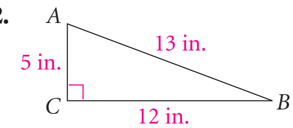
GO for Help Skills Handbook, p. 753

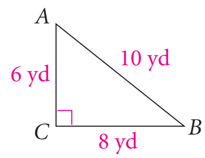
Square the lengths of the sides of each triangle. What do you notice?

1. 

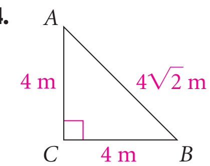
1. $3^2 + 4^2 = 5^2$

2. $5^2 + 12^2 = 13^2$

2. 

3. 

$6^2 + 8^2 = 10^2$

4. 

$4^2 + 4^2 = (4\sqrt{2})^2$

New Vocabulary • Pythagorean triple

1 The Pythagorean Theorem

The well-known right triangle relationship called the Pythagorean Theorem is named for Pythagoras, a Greek mathematician who lived in the sixth century B.C. We now know that the Babylonians, Egyptians, and Chinese were aware of this relationship before its discovery by Pythagoras.

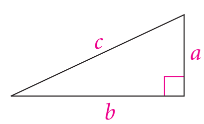
There are many proofs of the Pythagorean Theorem. You will see one proof in Exercise 48 and others later in the book.

Key Concepts

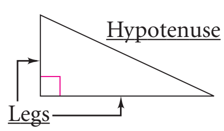
Theorem 8-1 Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$



Vocabulary Tip



A **Pythagorean triple** is a set of nonzero whole numbers a, b, and c that satisfy the equation $a^2 + b^2 = c^2$. Here are some common Pythagorean triples.

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25

If you multiply each number in a Pythagorean triple by the same whole number, the three numbers that result also form a Pythagorean triple.

Differentiated Instruction Solutions for All Learners

Special Needs L1

As you read the Pythagorean Theorem together with the class, point out how much easier it is when stated algebraically rather than in words. Have students practice reciting "$a^2 + b^2 = c^2$."

learning style: verbal

Below Level L2

Before the lesson, list the squares of whole numbers less than 20. Also review how to simplify a radical expression.

learning style: verbal

8-1

1. Plan

Objectives

1. To use the Pythagorean Theorem
2. To use the Converse of the Pythagorean Theorem

Examples

1. Pythagorean Triples
2. Using Simplest Radical Form
3. Real-World Connection
4. Is It a Right Triangle?
5. Classifying Triangles as Acute, Obtuse, or Right

Professional Development

Math Background

Some mathematical ideas assumed to be true have yet to be proved, such as Goldbach's conjecture: *Every even number greater than 2 can be expressed as the sum of two prime numbers.* Although several ancient cultures postulated the Pythagorean Theorem and used it to measure distances, the first proof of it was attributed by Euclid to Pythagoras. The distance formula is a coordinate form of the Pythagorean Theorem, which is the foundation of all trigonometric functions.

More Math Background: p. 414C

Lesson Planning and Resources

See p. 414E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to: Skills Handbook, p. 753

2. Teach

Guided Instruction

1 EXAMPLE Teaching Tip

Let students know that Pythagorean triples often appear on standardized tests.

2 EXAMPLE Error Prevention

Some students may assume that the legs are always the known quantities. Point out that c is always the hypotenuse when applying the formula $a^2 + b^2 = c^2$ to a right triangle.

3 EXAMPLE Technology Tip

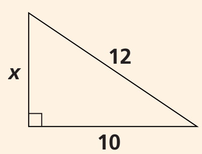
Students may wonder why they are asked to use a calculator in some exercises but not in other similar exercises. Tell them that real-world applications typically require decimal answers. Point out that radicals are exact, so they are preferred when exercises are of a purely mathematical nature.

PowerPoint

Additional Examples

1 A right triangle has legs of length 16 and 30. Find the length of the hypotenuse. Do the lengths of the sides form a Pythagorean triple? **34; yes**

2 Find the value of x. Leave your answer in simplest radical form.



$2\sqrt{11}$

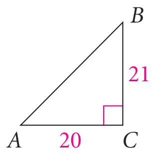
3 A baseball diamond is a square with 90-ft sides. Home plate and second base are at opposite vertices of the square. About how far is home plate from second base? **about 127 ft**

Test-Taking Tip

Memorizing the common Pythagorean triples, like those at the bottom of p. 417, can help you solve problems more quickly.

1 EXAMPLE Pythagorean Triples

Find the length of the hypotenuse of $\triangle ABC$. Do the lengths of the sides of $\triangle ABC$ form a Pythagorean triple?



$a^2 + b^2 = c^2$	**Use the Pythagorean Theorem.**
$21^2 + 20^2 = c^2$	**Substitute 21 for *a* and 20 for *b*.**
$441 + 400 = c^2$	**Simplify.**
$841 = c^2$	
$c = 29$	**Take the square root.**

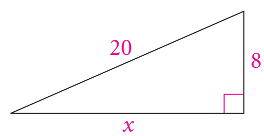
The length of the hypotenuse is 29. The lengths of the sides, 20, 21, and 29, form a Pythagorean triple because they are whole numbers that satisfy $a^2 + b^2 = c^2$.

Quick Check 1 A right triangle has a hypotenuse of length 25 and a leg of length 10. Find the length of the other leg. Do the lengths of the sides form a Pythagorean triple?
$5\sqrt{21}$; no

In some cases, you will write the length of a side in simplest radical form.

2 EXAMPLE Using Simplest Radical Form

Algebra Find the value of x. Leave your answer in simplest radical form (page 390).

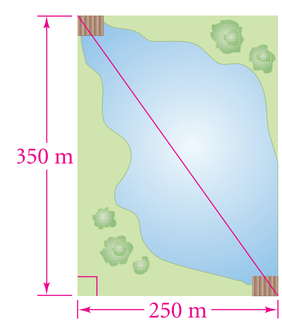


$a^2 + b^2 = c^2$	**Pythagorean Theorem**
$8^2 + x^2 = 20^2$	**Substitute.**
$64 + x^2 = 400$	**Simplify.**
$x^2 = 336$	**Subtract 64 from each side.**
$x = \sqrt{336}$	**Take the square root.**
$x = \sqrt{16(21)}$	**Simplify.**
$x = 4\sqrt{21}$	

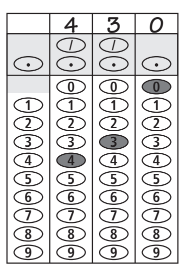
Quick Check 2 The hypotenuse of a right triangle has length 12. One leg has length 6. Find the length of the other leg. Leave your answer in simplest radical form.
$6\sqrt{3}$

3 EXAMPLE Real-World Connection

Gridded Response The Parks Department rents paddle boats at docks near each entrance to the park. To the nearest meter, how far is it to paddle from one dock to the other?



$a^2 + b^2 = c^2$	**Pythagorean Theorem**
$250^2 + 350^2 = c^2$	**Substitute.**
$185{,}000 = c^2$	**Simplify.**
$c = \sqrt{185{,}000}$	**Take the square root.**
$c = 430.11626$	**Use a calculator.**



It is 430 m from one dock to the other.

Quick Check 3 **Critical Thinking** When you want to know how far you have to paddle a boat, why is an approximate answer more useful than an answer in simplest radical form?
You want to know the nearest whole number value, which may not be apparent in a radical expression.

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
Have students describe how a triangle whose sides form a Pythagorean triple and a triangle whose sides are a multiple of that triple are related. Students should recognize that they are similar triangles.
learning style: verbal

English Language Learners ELL
Review the term *converse*, using the Pythagorean Theorem and its converse as an example. Then have students write the Pythagorean Theorem as a *biconditional* statement.
learning style: verbal

2 The Converse of the Pythagorean Theorem

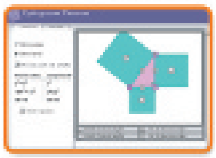
You can use the Converse of the Pythagorean Theorem to determine whether a triangle is a right triangle. You will prove Theorem 8-2 in Exercise 58.

Key Concepts

Theorem 8-2 **Converse of the Pythagorean Theorem**

If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

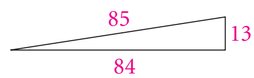
Online active math



For: Pythagorean Activity
Use: Interactive Textbook, 8-1

4 EXAMPLE Is It a Right Triangle?

Is this triangle a right triangle?



$c^2 \overset{?}{=} a^2 + b^2$

$85^2 \overset{?}{=} 13^2 + 84^2$ **Substitute the greatest length for c.**

$7225 \overset{?}{=} 169 + 7056$ **Simplify.**

$7225 = 7225$ ✓

$c^2 = a^2 + b^2$, so the triangle is a right triangle.

Quick Check 4 A triangle has sides of lengths 16, 48, and 50. Is the triangle a right triangle? **no**

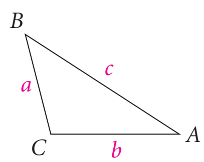
You can also use the squares of the lengths of the sides of a triangle to find whether the triangle is acute or obtuse. The following two theorems tell how.

Key Concepts

Theorem 8-3

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, the triangle is obtuse.

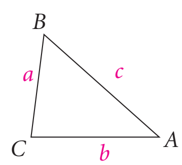
If $c^2 > a^2 + b^2$, the triangle is obtuse.

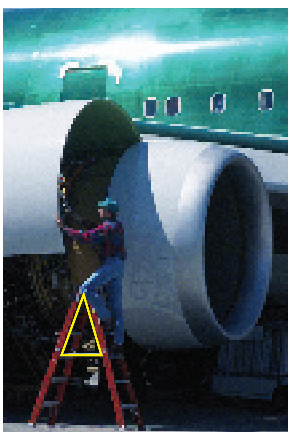


Theorem 8-4

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, the triangle is acute.

If $c^2 < a^2 + b^2$, the triangle is acute.





Real-World Connection

The length to the brace along each leg is 36 in. The brace is 26 in. long to guarantee that the triangle is acute.

5 EXAMPLE Classifying Triangles as Acute, Obtuse, or Right

Classify the triangle whose side lengths are 6, 11, and 14 as acute, obtuse, or right.

$14^2 \overset{?}{=} 6^2 + 11^2$ **Compare c^2 to $a^2 + b^2$. Substitute the greatest length for c.**

$196 \overset{?}{=} 36 + 121$

$196 > 157$

Since $c^2 > a^2 + b^2$, the triangle is obtuse.

Quick Check 5 A triangle has sides of lengths 7, 8, and 9. Classify the triangle by its angles. **acute**

Guided Instruction

Technology Tip

Have students use geometry software to explore and demonstrate the theorems *If* $c^2 > a^2 + b^2$, *the triangle is obtuse* and *If* $c^2 < a^2 + b^2$, *the triangle is acute.* Direct students to keep *a* and *b* constant while manipulating *c* by altering the angle opposite *c*.

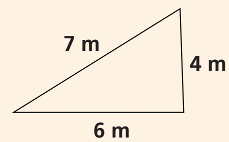
5 EXAMPLE Error Prevention

Remind students that *c* must be the longest side of the triangle for the comparison of c^2 and $a^2 + b^2$ to give a valid triangle classification. Also, students should use the Triangle Inequality Theorem to check that $a + b > c$ so that the side lengths form a triangle.

PowerPoint

Additional Examples

4 Is this triangle a right triangle?



no

5 The numbers represent the lengths of the sides of a triangle. Classify each triangle as acute, obtuse, or right.

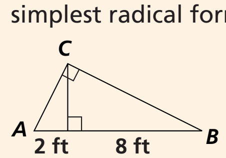
a. 15, 20, 25 **right**
b. 10, 15, 20 **obtuse**

Resources

- Daily Notetaking Guide 8-1 L3
- Daily Notetaking Guide 8-1—Adapted Instruction L1

Closure

The area of $\triangle ABC$ is 20 ft^2. Find *AC* and *BC*. Leave your answer in simplest radical form.



$AC = 2\sqrt{5}$ ft; $BC = 4\sqrt{5}$ ft

3. Practice

Assignment Guide

1 A B 1-17, 27-29, 32, 34-39, 48-53

2 A B 18-26, 30, 31, 33, 40-47

C Challenge 54-58

Test Prep 59-64
Mixed Review 65-73

Homework Quick Check

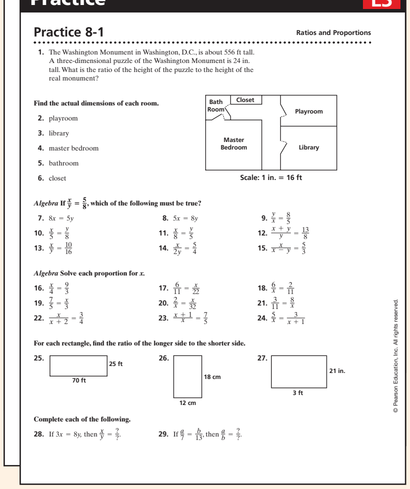
To check students' understanding of key skills and concepts, go over Exercises 16, 18, 30, 36, 50.

Exercises 14, 15 These exercises anticipate the special right triangle relationships in Lesson 8-2. Ask: *What is the ratio* a : b : c *in each triangle?* **1 : 1 : $\sqrt{2}$**

Exercises 21–26 In only some of the exercises do the first two lengths represent *a* and *b*. Remind students to compare the sum of the squares of the two smaller lengths with the square of the greatest length.

Differentiated Instruction Resources

GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Adapted Practice L1
Practice L3



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

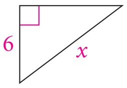
Practice and Problem Solving

A Practice by Example

GO for Help

Example 1 (page 418)

Algebra **Find the value of *x*.**

1. 

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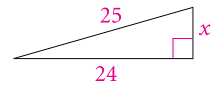
2. 

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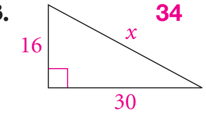
3. 

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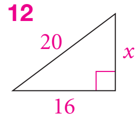
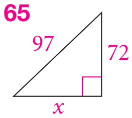
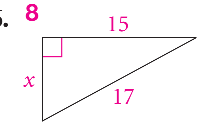
4. **12** 

5. **65** 

6. **8** 

Does each set of numbers form a Pythagorean triple? Explain.

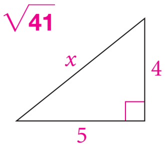
7. 4, 5, 6 **no; $4^2 + 5^2 \neq 6^2$**

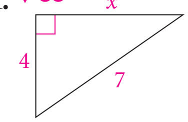
8. 10, 24, 26 **yes; $10^2 + 24^2 = 26^2$**

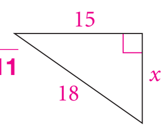
9. 15, 20, 25 **yes; $15^2 + 20^2 = 25^2$**

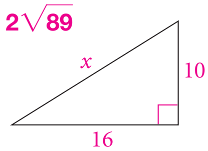
Example 2 (page 418)

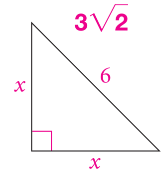
Algebra **Find the value of *x*. Leave your answer in simplest radical form.**

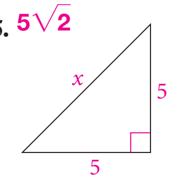
10. **$\sqrt{41}$** 

11. **$\sqrt{33}$** 

12. **$3\sqrt{11}$** 

13. **$2\sqrt{89}$** 

14. **$3\sqrt{2}$** 

15. **$5\sqrt{2}$** 

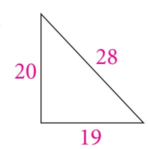
Example 3 (page 418)

16. **Home Maintenance** A painter leans a 15-ft ladder against a house. The base of the ladder is 5 ft from the house.
a. To the nearest tenth of a foot, how high on the house does the ladder reach? **14.1 ft**
b. The ladder in part (a) reaches too high on the house. By how much should the painter move the ladder's base away from the house to lower the top by 1 ft? **about 2.3 ft**

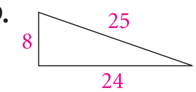
17. A walkway forms the diagonal of a square playground. The walkway is 24 m long. To the nearest tenth of a meter, how long is a side of the playground? **17.0 m**

Example 4 (page 419)

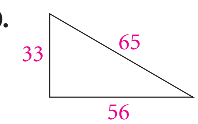
Is each triangle a right triangle? Explain.

18. 

no; $19^2 + 20^2 \neq 28^2$

19. 

no; $8^2 + 24^2 \neq 25^2$

20. 

yes; $33^2 + 56^2 = 65^2$

Example 5 (page 419)

The lengths of the sides of a triangle are given. Classify each triangle as acute, right, or obtuse.

21. 4, 5, 6 **acute**

22. 0.3, 0.4, 0.6 **obtuse**

23. 11, 12, 15 **acute**

24. $\sqrt{3}$, 2, 3 **obtuse**

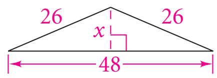
25. 30, 40, 50 **right**

26. $\sqrt{11}$, $\sqrt{7}$, 4 **acute**

B Apply Your Skills

Algebra **Find the value of x. Leave your answer in simplest radical form.**

27.



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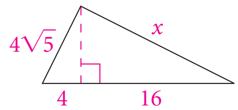
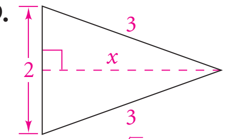
28.



$8\sqrt{5}$

29.



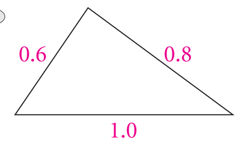
$2\sqrt{2}$

30. Writing Each year in an ancient land, a large river overflowed its banks, often destroying boundary markers. The royal surveyors used a rope with knots at 12 equal intervals to help reconstruct boundaries. Explain how a surveyor could use this rope to form a right angle. (*Hint:* Use the Pythagorean triple 3, 4, 5.)

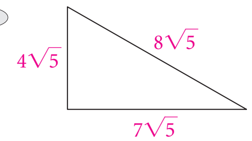
30. Answers may vary. Sample: Have three people hold the rope 3 units, 4 units, and 5 units apart in the shape of a triangle.

31. Multiple Choice Which triangle is *not* a right triangle? **B**

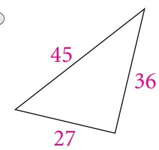
(A)



(B)



(C)



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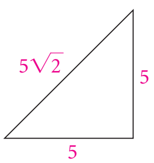
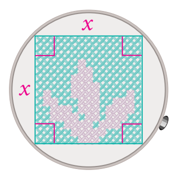


GO for Help

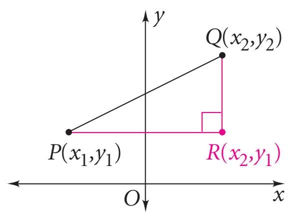
For a guide to solving Exercise 32, see p. 424.

32. Embroidery You want to embroider a square design. You have an embroidery hoop with a 6 in. diameter. Find the largest value of x so that the entire square will fit in the hoop. Round to the nearest tenth. **4.2 in.**



33. In parallelogram $RSTW$, $RS = 7$, $ST = 24$, and $RT = 25$. Is $RSTW$ a rectangle? Explain.
Yes; $7^2 + 24^2 = 25^2$, so $\angle RST$ is a rt. $\angle$.

Proof **34. Coordinate Geometry** You can use the Pythagorean Theorem to prove the Distance Formula. Let points $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the endpoints of the hypotenuse of a right triangle.



a. Write an algebraic expression to complete each of the following:
$PR = \blacksquare$ and $QR = \blacksquare$. **$|x_2 - x_1|$; $|y_2 - y_1|$**

b. By the Pythagorean Theorem, $PQ^2 = PR^2 + QR^2$. Rewrite this statement substituting the algebraic expressions you found for PR and QR in part (a).

34b. $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

c. Complete the proof by taking the square root of each side of the equation that you wrote in part (b). **$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$**

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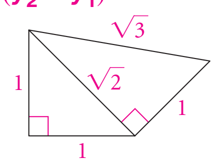
35. Constructions Explain how to construct a segment of length $\sqrt{n}$, where n is any positive integer, and you are given a segment of length 1. (*Hint:* See the diagram.) **See margin.**



Find a third whole number so that the three numbers form a Pythagorean triple.

36. 20, 21 **29** **37.** 14, 48 **50** **38.** 13, 85 **84** **39.** 12, 37 **35**

Error Prevention!

Exercise 28 Students may think the triangle with side lengths x, $4\sqrt{5}$, and 20 is a right triangle. Point out that there is no right angle symbol in the large triangle. Students must use the Pythagorean Theorem twice, first to find the side of the smallest triangle, and then to find the hypotenuse of the triangle with base 16.

Exercise 31 Show students how to use Pythagorean triples to check for right triangles. For answer choice A, they can multiply each side by 10 to get sides of 6, 8 and 10. They should recognize this as a multiple of a 3, 4, 5 triangle. Similarly, by dividing each side in answer choice B by $\sqrt{5}$, students can recognize that the triangle cannot be a right triangle.

Exercise 44 Some students may be unfamiliar with the terms *embroider* and *embroidery hoop.* Ask a volunteer to bring embroidery materials and an embroidery hoop to class and demonstrate how to use the hoop.

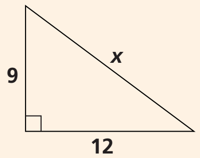
35. Answers may vary. Sample: Using 2 segments of length 1, construct the hyp. of the right $\triangle$ formed by these segments. Using the hyp. found as one leg and a segment of length 1 as the other leg, construct the hyp. of the $\triangle$ formed by those legs. Continue this process until constructing a hypotenuse of length $\sqrt{n}$.

4. Assess & Reteach

PowerPoint

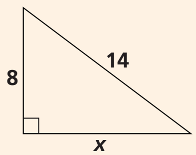
Lesson Quiz

1. Find the value of x.



15

2. Find the value of x. Leave your answer in simplest radical form.



$2\sqrt{33}$

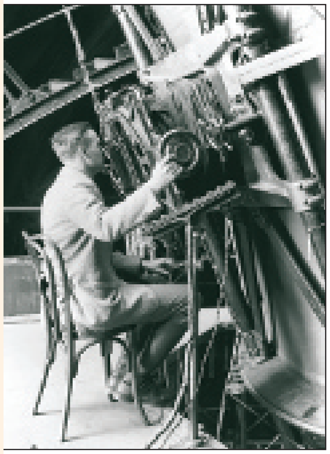
3. The town of Elena is 24 mi north and 8 mi west of Holberg. A train runs on a straight track between the two towns. How many miles does it cover? Round your answer to the nearest whole number. 25 mi

4. The lengths of the sides of a triangle are 5 cm, 8 cm, and 10 cm. Is it acute, right, or obtuse? obtuse

Alternative Assessment

Have students use the Pythagorean Theorem to find the length of the diagonal of their notebook paper and explain in writing how the Pythagorean Theorem was used. Then have them measure the diagonal to confirm the length found using the Pythagorean Theorem.

48. $\frac{r}{a} = \frac{a}{c}$ and $\frac{q}{b} = \frac{b}{c}$. So $a^2 = rc$ and $b^2 = qc$. $a^2 + b^2 = rc + qc = (r + q)c = c^2$



Real-World Connection

Research by Edwin Hubble (1889–1953), here guiding a telescope in 1923, led to the Big Bang Theory of the formation of the universe.

53a. Answers may vary. Sample: $n = 6$; 12, 35, 37

Challenge

53c. $(2n)^2 + (n^2 - 1)^2$
$= 4n^2 + n^4 - 2n^2 + 1$
$= n^4 + 2n^2 + 1$
$= (n^2 + 1)^2$

Find integers j and k so that (a) the two given integers and j represent the lengths of the sides of an acute triangle and (b) the two given integers and k represent the lengths of the sides of an obtuse triangle. 40–47. Answers may vary. Samples are given.

40. 4, 5 6; 7 **41.** 2, 4 4; 5 **42.** 6, 9 8; 11 **43.** 5, 10 11; 12

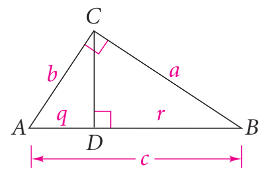
44. 6, 7 8; 10 **45.** 9, 12 14; 16 **46.** 8, 17 18; 19 **47.** 9, 40 39; 42

Proof **48.** Prove the Pythagorean Theorem.

Given: $\triangle ABC$ is a right triangle

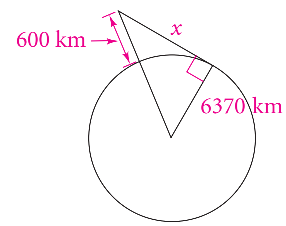
Prove: $a^2 + b^2 = c^2$

(*Hint*: Begin with proportions suggested by Theorem 7-3 or its corollaries.)



49. Astronomy The Hubble Space Telescope is orbiting Earth 600 km above Earth's surface. Earth's radius is about 6370 km. Use the Pythagorean Theorem to find the distance x from the telescope to Earth's horizon. Round your answer to the nearest ten kilometers.

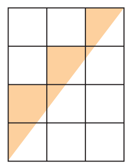
2830 km



not to scale

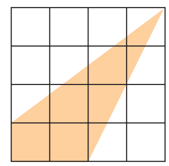
The figures below are drawn on centimeter grid paper. Find the perimeter of each shaded figure to the nearest tenth.

50. GPS



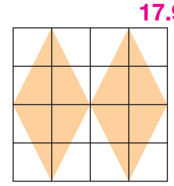
12 cm

51.



12.5 cm

52.



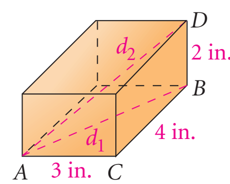
17.9 cm

53. a. The ancient Greek philosopher Plato used the expressions $2n$, $n^2 - 1$, and $n^2 + 1$ to produce Pythagorean triples. Choose any integer greater than 1. Substitute for n and evaluate the three expressions. $12^2 + 35^2 = 37^2$

b. Verify that your answers to part (a) form a Pythagorean triple.

c. Show that, in general, $(2n)^2 + (n^2 - 1)^2 = (n^2 + 1)^2$ for any n.

54. Geometry in 3 Dimensions The box at the right is a rectangular solid.

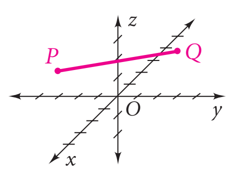


a. Use $\triangle ABC$ to find the length d_1 of the diagonal of the base. 5 in.

b. Use $\triangle ABD$ to find the length d_2 of the diagonal of the box. $\sqrt{29}$

c. You can generalize the steps in parts (a) and (b). Use the facts that $AC^2 + BC^2 = d_1^2$ and $d_1^2 + BD^2 = d_2^2$ to write a one-step formula to find d_2. $d_2 = \sqrt{BD^2 + AC^2 + BC^2}$

d. Use the formula you wrote to find the length of the longest fishing pole you can pack in a box with dimensions 18 in., 24 in., and 16 in. 34 in.



Geometry in 3 Dimensions **Points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ at the left are points in a three-dimensional coordinate system. Use the following formula to find PQ. Leave your answer in simplest radical form.**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

55. $P(0, 0, 0)$, $Q(1, 2, 3)$ $\sqrt{14}$

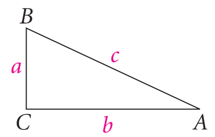
56. $P(0, 0, 0)$, $Q(-3, 4, -6)$ $\sqrt{61}$

57. $P(-1, 3, 5)$, $Q(2, 1, 7)$ $\sqrt{17}$

Proof **58.** Use the plan and write a proof of Theorem 8-2, the Converse of the Pythagorean Theorem.

Given: $\triangle ABC$ with sides of length a, b, and c where $a^2 + b^2 = c^2$

Prove: $\triangle ABC$ is a right triangle.



Plan: Draw a right triangle (not $\triangle ABC$) with legs of lengths a and b. Label the hypotenuse x. By the Pythagorean Theorem, $a^2 + b^2 = x^2$. Use substitution to compare the lengths of the sides of your triangle and $\triangle ABC$. Then prove the triangles congruent. **See margin.**

Test Prep

Gridded Response

59. The lengths of the legs of a right triangle are 17 m and 20 m. To the nearest tenth of a meter, what is the length of the hypotenuse? **26.2**

60. The hypotenuse of a right triangle is 34 ft. One leg is 16 ft. Find the length of the other leg in feet. **30**

61. What whole number forms a Pythagorean triple with 40 and 41? **9**

62. The two shorter sides of an obtuse triangle are 20 and 30. What is the least whole number length possible for the third side? **37**

63. Each leg of an isosceles right triangle has measure 10 cm. To the nearest tenth of a centimeter, what is the length of the hypotenuse? **14.1**

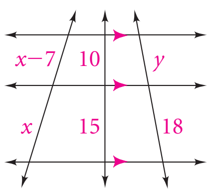
64. The legs of a right triangle have lengths 3 and 4. What is the length, to the nearest tenth, of the altitude to the hypotenuse? **2.7**

Mixed Review

GO for Help

Lesson 7-5

For the figure at the right, complete the proportion.



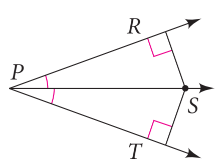
65. $\frac{10}{\blacksquare} = \frac{y}{18}$ **15**

66. $\frac{\blacksquare}{x} = \frac{y}{18}$ **x − 7**

67. Find the values of x and y. **x = 21, y = 12**

Lesson 5-2

In the second figure, $\overrightarrow{PS}$ bisects $\angle RPT$. Solve for each variable. Then find RS.



68. $RS = 2x + 19, ST = 7x - 16;\ x = \blacksquare, RS = \blacksquare$ **7; 33**

69. $RS = 2(7y - 11), ST = 5y + 5;\ y = \blacksquare, RS = \blacksquare$ **3; 20**

Lesson 4-1

$\triangle PQR \cong \triangle STV$. Solve for each variable.

70. $m\angle P = 4w + 5, m\angle S = 6w - 15$ **10**

71. $RQ = 10y - 6, VT = 5y + 9$ **3**

72. $m\angle T = 2x - 40, m\angle Q = x + 10$ **50**

73. $PR = 2z + 3, SV = 4z - 11$ **7**

Test Prep

A sheet of blank grids is available in the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 465
- Test-Taking Strategies, p. 460
- Test-Taking Strategies with Transparencies

58. Draw right $\triangle FDE$ with legs $\overline{DE}$ of length a and $\overline{EF}$ of length b, and hyp. of length x. Then $a^2 + b^2 = x^2$ by the Pythagorean Thm. We are given $\triangle ABC$ with sides of length a, b, c and $a^2 + b^2 = c^2$. By subst., $c^2 = x^2$, so $c = x$. Since all side lengths of $\triangle ABC$ and $\triangle FDE$ are the same, $\triangle ABC \cong \triangle FDE$ by SSS. $\angle C \cong \angle E$ by CPCTC, so $m\angle C = 90$. Therefore, $\triangle ABC$ is a right $\triangle$.