Symmetry

What You’ll Learn
• To identify the type of symmetry in a figure

...And Why
To identify types of symmetry in real-life objects, as in Example 3.

Check Skills You’ll Need
To identify the type of symmetry in a figure

The regular octagon at the right is divided into eight congruent triangles. Find the image of the given point or segment for the given rotation or reflection.

1. point \( A \); a 90° rotation about the center
2. point \( H \); a 180° rotation about the center
3. \( AB \); a reflection in \( AE \)
4. \( GH \); a reflection in \( AE \)

New Vocabulary
• symmetry
• reflectional symmetry
• line symmetry
• rotational symmetry
• point symmetry

Identifying Types of Symmetry in Figures

A figure has symmetry if there is an isometry that maps the figure onto itself. If the isometry is the reflection of a plane figure, the figure has reflectional symmetry or line symmetry. One half of the figure is a mirror image of its other half. Fold the figure along the line of symmetry and the halves match exactly.

The image of a sculpture at the right has reflectional symmetry about a vertical line down the middle of the face.

It is possible for a figure to have more than one line of symmetry.

1. **EXAMPLE**  
   **Identifying Lines of Symmetry**

   Draw all lines of symmetry for a regular hexagon.

   **Solution:**
   - Draw a regular hexagon.
   - Then draw lines on the hexagon that make mirror-image congruent halves.

   There are 6 lines of symmetry.

Quick Check
1. Draw a rectangle and all of its lines of symmetry.
A figure that has **rotational symmetry** is its own image for some rotation of 180° or less. A figure that has **point symmetry** has 180° rotational symmetry. A square has 90° and 180° rotational symmetry with the center of rotation at the center of the square. Thus, a square also has point symmetry.

### Identifying Rotational Symmetry

Judging from appearance, tell whether each triangle has rotational symmetry. If so, give the angle of rotation.

**a.** The equilateral triangle has rotational symmetry. The angle of rotation is 120°.

This isosceles triangle does not have rotational symmetry.

**b.** Does the figure have point symmetry? **yes**

Three-dimensional objects can have various types of symmetry, including rotational symmetry about a line and reflectional symmetry in a plane.

### Real-World Connection

**Symmetric Design** Tell whether each object has rotational symmetry about a line and/or reflectional symmetry in a plane.

**a.** The paddle has both rotational and reflectional symmetry.

**b.** The cup has reflectional symmetry.

Tell whether the umbrella has rotational symmetry about a line and/or reflectional symmetry in a plane.

**rotational and reflectional symmetry**
Tell what type(s) of symmetry each figure has. If it has line symmetry, sketch
the figure and the line(s) of symmetry. If it has rotational symmetry, state the
angle of rotation.

1. 2. 3. 4.

5. 6. 7. 8.

9. 10. 11. 12.

Tell whether each three-dimensional object has rotational symmetry about a line
and/or reflectional symmetry in a plane.

13. 14. 15. 16.

Draw each quadrilateral. Then draw all of its lines of symmetry.

13. rhombus 14. kite 15. square 16. parallelogram

Tell whether each three-dimensional object has rotational symmetry about a line
and/or reflectional symmetry in a plane.

17. 18.


19. CODE, HOOD, DOCK
20. TOMATO, HOAX, WAXY

21b. Sample: Greek;
Greek alphabet has
more letters with at
least one kind of
symmetry and more
letters with multiple
symmetries.

21a. Open-Ended
A. Alphabets
Copy the chart. Use
it to classify the letters of the
English and Greek alphabets
below. You will list some letters
in more than one category.
B. Which alphabet can you say
is more symmetrical? Explain.

<table>
<thead>
<tr>
<th>Language</th>
<th>Horizontal Line</th>
<th>Vertical Line</th>
<th>Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>English</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greek</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Greek: Α Β Γ Δ Ε Ζ Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ Σ Τ Υ Φ ψ ω

494 Chapter 9 Transformations
Tell what type(s) of symmetry each image has. For line symmetry, sketch the image and the line(s) of symmetry. For rotational symmetry, state the angle of rotation.

22–23. Check students’ sketches.

24. Open-Ended  The equation \( |10 - 1| = 93 \) is not only true, but also symmetrical (horizontally). Write four other equations or inequalities that are both true and symmetrical. \( \text{See left.} \)

25. reflectional; rotational
26. reflectional
29. reflectional; rotational
31. reflectional, rotational

33. Is the line that contains the bisector of an angle a line of symmetry of the angle? Explain. \( \text{See margin.} \)
34. Is the line that contains the bisector of an angle of a triangle a line of symmetry of the triangle? Explain. \( \text{See left.} \)
35. Is a bisector of a segment a line of symmetry of the segment? Explain. \( \text{See margin.} \)

36. Multiple Choice Which statement is true about the figure at the right? \( \text{D} \) It has no rotational symmetry.
   \( \text{E} \) It has no reflectional symmetry.
   \( \text{F} \) It has rotational symmetry with an angle of rotation of 45°.
   \( \text{G} \) It has reflectional symmetry with six lines of symmetry.

Coordinate Geometry A figure has a vertex at \((3, 4)\). If the figure has the given type of symmetry, state the coordinates of another vertex of the figure.
37. line symmetry about the \( y \)-axis \((-3, 4)\) 38. line symmetry about the \( x \)-axis
39. point symmetry about the origin 40. line symmetry about the line \( y = x \) \((-3, -4)\)

Coordinate Geometry Graph each equation. Describe the symmetry of each graph.
41. \( y = x \) 42. \( y = x^2 \) 43. \( x = y^2 \) 44. \( x^2 + y^2 = 9 \)
45. \( y = (x + 2)^2 \) 46. \( y = x^3 \) 47. \( y = |x| \) 48. \( x = |y| \)

33. Yes; the bisector divides the \( \angle \) into 2 \( \cong \) \( \triangle \) with one side of the \( \angle \) being the reflection of the other.
35. Not necessarily; the bisector divides the segment into 2 \( \cong \) parts but one part cannot be the reflection of the other unless the bisector is the \( \perp \) bisector.
For each three-dimensional figure, draw a net that has rotational symmetry and a net that has 1, 2, or 4 lines of symmetry. 49–50. See back of book.

49. Which figure does NOT have rotational symmetry? B

A. B. C. D.

50. Which figure, in general, has exactly two lines of symmetry? J

F. pentagon G. circle H. square J. rectangle

51. Which quadrilateral has rotational symmetry but not reflectional symmetry? D

A. nonisosceles trapezoid B. kite C. rhombus D. parallelogram

52. What is the smallest angle through which you can rotate a regular hexagon onto itself? G

F. 30° G. 60° H. 90° J. 120°

53. Does a regular octagon have a. line symmetry? If so, how many lines of symmetry does it have? b. rotational symmetry? If so, what is the angle of rotation?

54. Use the figure at the right to answer the questions below.

a. Does the figure have rotational symmetry? If so, identify the angle of rotation. a-b. See margin.

b. Does the figure have reflectional symmetry? If so, how many lines of symmetry does it have?

55. Which capital letters of the alphabet are rotation images of themselves? Draw each letter and give an angle of rotation (< 360°).

H, I, O, X, N, S, Z

The blue figure is a transformation image of the black figure. Does the transformation appear to be an isometry? Explain. 58–60. See left.

56. Three vertices of an isosceles trapezoid are (−2, 1), (1, 4), and (4, 4). Find possible pairs of coordinates for the fourth vertex. (−2, −2) or (7, 1)