Chapter Review

Vocabulary Review

center of a regular polygon (p. 484)
composition (p. 472)
dilation (p. 498)
enlargement (p. 498)
glide reflection (p. 508)
glide reflectional symmetry (p. 516)
image (p. 470)
rotate (p. 470)
rotational symmetry (p. 502)
symmetry (p. 492)
tessellation (p. 515)
tiling (p. 515)
transformation (p. 470)
translational symmetry (p. 516)

To complete each definition, find the appropriate word in the second column.

1. A(n) __________ is a change in position, shape, or size of a figure.  
   A. dilation  
   F. translation

2. A(n) __________ is a transformation in which the preimage and its image are congruent.  
   B. glide reflection  
   D. isometry

3. A __________ is an isometry in which a figure and its image have opposite orientations.  
   C. tessellation  
   E. reflection

4. A __________ is an isometry in which all points of a figure move the same distance in the same direction.  
   G. translation

5. A(n) __________ is a translation followed by a reflection in a line parallel to the translation vector.  
   B. glide reflection  
   E. reflection

6. A(n) __________ is a repeating pattern of figures that completely covers a plane, without gaps or overlaps.  
   F. transformation  
   C. tessellation

7. A(n) __________ is a transformation that proportionally reduces or enlarges a figure.  
   G. translation  
   A. dilation

Skills and Concepts

9-1 and 9-2 Objectives

- To identify isometries
- To find translation images of figures
- To find reflection images of figures

A transformation of a geometric figure is a change in its position, shape, or size. An isometry is a transformation in which the preimage and image are congruent. A transformation maps a figure onto its image.

A translation is an isometry that maps all points of a figure the same distance in the same direction. A translation is an isometry that does not change orientation.

The second diagram shows a reflection of B to B' across line r. A reflection is an isometry in which a figure and its image have opposite orientations.

A composition of transformations is a combination of two or more transformations. Each transformation is performed on the image of the preceding transformation.
8. Use matrices to find the image of each triangle for the given translation.

8. \( \triangle ABC \) with vertices \( A(5, 9), B(6, 3), C(1, 2) \); translation: \((x, y) \rightarrow (x + 2, y + 3)\)

9. \( \triangle RST \) with vertices \( R(-4, -1), S(-6, 1), T(-10, 8) \); translation: \((x, y) \rightarrow (x - 4, y + 7)\)

Find a single translation that has the same effect as each composition.

10. \((x, y) \rightarrow (x - 5, y + 7)\) followed by \((x, y) \rightarrow (x + 3, y - 6)\)

11. \((x, y) \rightarrow (x + 10, y - 9)\) followed by \((x, y) \rightarrow (x + 1, y + 5)\)

Given points \( A(6, 4), B(-2, 1), \) and \( C(5, 0) \), draw \( \triangle ABC \) and its reflection image in each line. 12–14. See margin.

12. the \( x \)-axis
13. \( x = 4 \)
14. \( y = x \)

The diagram shows a rotation of point \( V \) about point \( R \) through \( 45^\circ \). A rotation is an isometry that does not change orientation.

Copy each figure and point \( P \). Draw the image of each figure for the given rotation about \( P \). Label the vertices of the image.

15. \( 180^\circ \) rotation about \( P \)
16. \( 60^\circ \) rotation about \( P \)
17. \( 90^\circ \) rotation about \( P \)

Find the image of each point for a \( 90^\circ \) rotation about the origin.

18. \((5, 2)\) \(\rightarrow (2, 5)\)
19. \((0, 3)\) \(\rightarrow (3, 0)\)
20. \((-4, 1)\) \(\rightarrow (1, -4)\)
21. \((7, 0)\) \(\rightarrow (0, 7)\)
22. \((-2, -8)\) \(\rightarrow (8, -2)\)

A figure has symmetry if there is an isometry that maps the figure onto itself. A plane figure has reflectional symmetry, or line symmetry, if one half of the figure is a mirror image of its other half. A figure that has rotational symmetry is its own image for some rotation of \( 180^\circ \) or less. A figure that has point symmetry has \( 180^\circ \) rotational symmetry.

Tell what type(s) of symmetry each figure has. If it has rotational symmetry, state the angle of rotation.

23. rotational; \( 72^\circ \)
24. reflectional
Chapter Review

A tessellation, or tiling, is a repeating pattern of figures that completely covers a plane, without gaps or overlaps. A tessellation can have translational symmetry if there is a translation that maps the tessellation onto itself. If a tessellation can be mapped onto itself by a glide reflection, then the tessellation has glide reflectional symmetry.

For each tessellation, (a) identify a transformation and the repeating figures, and (b) list the symmetries. 35-36. See back of book.

35. 

36. 

A dilation has center (0, 0). Find the image of each point for the scale factor given.

25. A(0, 3); 4

26. B(−2, 6); 0.5

27. C(1.5, −2); 10

Find the image of each set of points for a dilation with center and the scale factor given.

28. M(−3, 4), A(−6, −1), T(0, 0), H(3, 2); scale factor 5

29. F(−4, 0), U(5, 0), N(−2, −5); scale factor \( \frac{1}{2} \)

A composition of reflections in two parallel lines is a translation. A composition of reflections in two intersecting lines is a rotation. A glide reflection is the composition of a glide (translation) and a reflection in a line parallel to the translation vector. The only four isometries are reflection, translation, rotation, and glide reflection.

For the figure at the left below, four isometry images are shown. Tell whether orientations are the same or opposite. Then classify the isometry. 30–33. See margin.

34. Triangle TAM has vertices T(0, 5), A(4, 1), and M(3, 6). Find the image of triangle TAM where the translation is \((x, y) \rightarrow (x − 4, y)\) and the reflection is in the line \(y = −2\).

\( T'(−4, −9), A'(0, −5), M'(−1, −10) \)
Chapter Test

Find the coordinates of the vertices of the image of $ABCD$ for each transformation.
1. reflection across the line $x = -4$
2. translation $(x, y) \rightarrow (x - 6, y + 8)$
3. rotation of 90° about the point $(0, 0)$
4. dilation centered at $(0, 0)$ with scale factor $\frac{2}{3}$
5. glide reflection with translation $(x, y) \rightarrow (x, y + 3)$ and reflection across the line $x = 0$
6. reflection across the line $y = x$
7. rotation of 270° about $(0, 0)$
8. dilation centered at the origin with scale factor 5
9. glide reflection with translation $(x, y) \rightarrow (x - 2, y)$ and reflection across the line $y = 5$
10. translation 3 units right and 1 unit down

What type of transformation has the same effect as each composition of transformations?
11. translation $(x, y) \rightarrow (x + 4, y)$ followed by a reflection across the line $y = -4$
12. translation $(x, y) \rightarrow (x + 4, y + 8)$ followed by $(x, y) \rightarrow (x - 2, y + 9)$
13. reflection across the line $y = 7$, and then across the line $y = 3$
14. reflection across the line $y = x$, and then across the line $y = 2x + 5$

Draw a figure that has each type of symmetry.
15. reflective
16. rotational
17. point
18. rotational, reflective
19. rotational, point, reflective

Does each letter tessellate? If so, sketch a tessellation. If not, explain why it cannot tessellate.

Find the image of $\triangle ABC$ for a dilation with center $(0, 0)$ and the scale factor given.

20. Writing Line $m$ intersects $\overline{UH}$ at $N$, and $UN = NH$. Must $H$ be the reflection image of $U$ across line $m$? Explain. See back of book.
21. Describe the symmetries of this tessellation. Copy a portion of the tessellation and draw any centers of rotational symmetry or lines of symmetry. See back of book.

Find the image of $\triangle ABC$ for a dilation with center $(0, 0)$ and the scale factor given.

22. 23. 24.

25. $A(-2, 2), B(2, -2), C(3, 4)$; scale factor 3
26. $A(0, 0), B(-3, 2), C(1, 7)$; scale factor $\frac{1}{2}$
27. The blue figure is a translation image of the red figure. Write a rule to describe the translation. $(x, y) \rightarrow (x + 3, y - 3)$

28. A dilation with center $(0, 0)$ and scale factor 2.5 maps $(4, -10)$ to $(a, b)$. Find the values of $a$ and $b$.
29. A dilation maps $\triangle LMN$ to $\triangle L'M'N'$. Find the missing values.

$L = 36$ ft, $LN = 26$ ft, and $MN = 45$ ft;
$L' = 9$ ft, $L'N' = \text{ft}$, and $M'N' = \text{ft}$;
scale factor $= \frac{6}{12} = 3$

30. A dilation with scale factor 4 maps square $A$ onto square $B$. The area of square $B$ is 25. Find the area of square $A$. $1.5625$ units$^2$

1. $A'(-11, 0), B'(-9, -2),$  
   $C'(-11, -5), D'(-15, -1)$
2. $A'(-3, 8), B'(-5, 6),$  
   $C'(-3, 3), D'(1, 7)$
3. $A'(0, 3), B'(2, 1), C'(5, 3),$  
   $D'(1, 7)$
4. $A'(2, 0), B'(\frac{5}{2}, -1\frac{1}{2}),$  
   $C'(2, -3\frac{1}{2}), D'(4\frac{2}{3}, -\frac{2}{3})$
5. $A'(-3, 3), B'(-1, 1),$  
   $C'(-3, -2), D'(-7, 2)$
6. $A'(0, 3), B'(-2, 1),$  
   $C'(-5, 3), D'(-1, 7)$
7. $A'(0, -3), B'(-2, -1),$  
   $C'(-5, -3), D'(-1, -7)$
8. $A'(15, 0), B'(5, -10),$  
   $C'(15, -25), D'(35, -5)$
9. $A'(1, 10), B'(-1, 12),$  
   $C'(1, 15), D'(5, 11)$
10. $A'(6, -1), B'(4, -3),$  
    $C'(6, -8), D'(10, -2)$
Reading Comprehension  Read the passage below. Then answer the questions on the basis of what is stated or implied in the passage.

Hanging a Picture  A picture has a wire from side to side across its back. Hang the picture from one hook and it can easily swing, or slide into a tilt. Use two hooks and the picture will hang level. Here is how to place two hooks on the wall to hang the picture level and precisely where you want it.

On your wall, mark two level points A and B where you want the top corners of the picture. For example, assume the back of a 21-in. wide picture is rigged as shown at the left and you want the two hooks 12 in. apart.

At the right, the matching diagrams in the upper corners locate points Q and R where hooks would hold the wire. Determine measurements as follows:

1. How to use one hook so that you can easily slide the picture to hang straight.
2. How to use two hooks so that the picture hangs straight and where you want it.
3. How to use one hook so that the picture can easily swing, or slide into a tilt.
4. How to use two hooks at the top corners.

1. From the passage, what should you learn about hanging a picture?
   - A. How to use one hook so that you can easily slide the picture to hang straight.
   - B. How to use two hooks so that the picture hangs straight and where you want it.
   - C. How to use one hook so that the picture can easily swing, or slide into a tilt.
   - D. How to use two hooks at the top corners.

2. How do you calculate QT?
   - F. PQ = RS
   - H. PT = SV + 2

3. How do you calculate TP?
   - A. AB = 6
   - C. AB = 12
   - D. AB = 10

4. Which theorem do you use to calculate QT?
   - G. 30°-60°-90° Triangle
   - H. 45°-45°-90° Triangle
   - I. Triangle Midsegment

5. How do you calculate CQ?
   - A. CQ = DR
   - B. CQ = 6
   - C. CQ = 8.3
   - D. CQ = QT = CT

6. What kind of quadrilateral is DSV? Justify your answer.
   - Rectangle; it has 4 right angles.
   - See margin.

Describe how to locate the hooks for hanging.

7. A picture is 30 in. wide. The hanging wire is 34 in. long, attached at the sides of the picture, 9 in. from the top. The hooks are 14 in. apart. See margin.

8. A circular mirror has diameter 22 in. The hanging wire is 28 in. long, attached at the endpoints of a diameter. The hooks are 10 in. apart. See margin.